MAN-BITES-DOG
BUSINESS CYCLES

KRISTOFFER P. NIMARK

Abstract. The newsworthiness of an event is partly determined by how unusual it is and this paper investigates the business cycle implications of this fact. We analyze the implications of information structures in which some types of signals are more likely to be observed after unusual events. Such signals may increase both uncertainty and disagreement among agents and when embedded in a simple business cycle model, can help us understand why we observe (i) large changes in macro economic aggregate variables without a correspondingly large change in underlying fundamentals (ii) persistent periods of high macroeconomic volatility and (iii) a positive correlation between absolute changes in macro variables and the cross-sectional dispersion of expectations as measured by survey data. These results are consequences of optimal updating by agents when the availability of some signals is positively correlated with tail-events. The model is estimated by likelihood based methods using individual survey responses and a quarterly time series of total factor productivity along with standard aggregate time series. The estimated model suggests that there have been episodes in recent US history when the impact on output of innovations to productivity of a given magnitude were more than three times as large compared to other times.

1. Introduction

A well-known journalistic dictum states that "*dog-bites-man* is not news, but *man-bites-dog* is news". That is, unusual events are more likely to be considered newsworthy than events that are commonplace. This paper investigates the business cycle implications of this aspect of news reporting. Particularly, we will demonstrate that a single and relatively simple mechanism can help us understand three features of business cycles. First, there can be large changes in aggregate variables like CPI inflation and GDP growth, but without an

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Address: CREI, Universitat Pompeu Fabra and Barcelona GSE, Ramon Trias Fargas 25-27, Barcelona 08005.
e-mail: knimark@crei.cat *web*: www.kris-nimark.net.

easily identifiable change in fundamentals of comparable magnitude. Second, there appears to be persistent episodes of increased macroeconomic volatility in the data. Third, measures of uncertainty as well as measures of cross-sectional dispersion of expectations are positively correlated with absolute magnitudes of changes in macro economic aggregates. These features can be explained by Bayesian agents optimally updating to signals that are more likely to be available about unusual events. The model can also help us understand a type of "crisis mentality" in which an intense media focus on the economy causes an increase in both agents' uncertainty and sensitivity to new information.

Conceptually, information about the current state of the world can be divided into at least three categories. What we may call *local information* is information that agents observe directly through their interactions in markets, e.g. through buying and selling goods or through participating in the labor market. A second type of information is what we may call *statistics.* Statistics are collected and summarized by (often government) organizations and made available to a broader public through web sites and printed media. Statistics are normally reported regardless of the realized values of the variable that they refer to and often according to a pre-specified schedule. A third type is information provided by the news media, such as newspapers and television programs. News media may be the main source of information for a large section of the general population (e.g. Blinder and Kruger 2004). One service that the news media provides is to select what events to report. This editorial function of the news media is necessary since it is simply not possible for a newspaper or a television news program to report all events that has occurred on a given day or during a given week. The man-bites-dog dictum referred to above suggests that more unusual events are more likely to be selected for dissemination and more unusual events are thus more likely to become *news.* This makes news different from statistics since whether an outcome of an event is available as news depends on the realized outcome of the event. In order to have a terminology that is distinct from the one used by the literature studying how information about future productivity affect the economy today (e.g. Beaudry and Portier 2006 and Jaimovich and Rebelo 2009) "news" in the sense meant here will be referred to as man-bites-dog signals.

A prime example of man-bites-dog news reporting is the *Movers* segment on Bloomberg Television. In a typical segment, the price movements of a few stocks are reported along with short statements on the probable causes of these movements. The stocks in question are a small sub-sample of all stocks traded and are selected on the basis of having had the largest price movements during the day. Unusually large price movements are thus more likely to be reported than more common price movements. Because of the way that stock prices are selected for inclusion in the *Movers* segment, the variance of a stock's price conditional on it being mentioned in the *Movers* segment is thus clearly larger than its unconditional variance.

The example of the *Movers* segment illustrates a more general point. When the availability of a signal depends on the realized value of the variable of interest, the *availability* of the signal is in itself informative. Below, we will prove this more formally in a general setting where agents want to form an estimate of a latent variable. There, it will be shown that man-bites-dog news selection introduces a form of conditional heteroscedasticity. From the agents' perspective, it is *as if* the variable is drawn from a distribution with relatively more probability mass in the tails when the man-bites-dog signal is available, compared to the variable's unconditional distribution. Since whether the man-bites-dog signal is available or not is a discrete event, this mechanism effectively splits the unconditional distribution of the latent variable into two distinct conditional distributions.

When signals are noisy, the information contained in the availability of a man-bites-dog signal will in general affect both the mean and variance of agents' posteriors beliefs. Using a simple static setting, it can be shown that agents' conditional expectations about a latent variable respond stronger to a man-bites-dog signal relative to a standard signal of the same precision. The intuition is straightforward: The availability of a man-bites-dog signal suggests that tail realizations are relatively more likely so agents are then willing to move their expectations about the latent variable further from the prior mean. When agents' actions depend on their expectations about the latent variable, actions will also respond stronger to a man-bites-dog signal than to a standard signal of the same precision.

Perhaps more surprising, observing a man-bites-dog signal can potentially make agents more uncertain about the latent variable. To understand this result, note that a man-bites-dog signal affects agents' posterior uncertainty through two different channels that work in opposite directions. The fact that the signal is available means that agents should redistribute probability mass towards the tails of the distribution since unusually large realizations are conditionally more likely when the man-bites-dog signal is available. This effect increases agents' uncertainty. But the signal also contains information about the realized value of the variable which decreases uncertainty. When the signal is sufficiently noisy, the increase in uncertainty from the first effect dominates and agents' posteriors beliefs have a higher variance when the man-bites-dog signal is available compared to when the signal is not available.

In a dynamic setting, a larger posterior uncertainty in period t translates into a larger prior uncertainty in period $t+1$. By embedding a man-bites-dog information structure in a simple business cycle model similar to that of Lorenzoni (2009), we show that the propagation of uncertainty through time endogenously generates periods of persistently higher volatility in output and inflation. The mechanism is the following. Agents in the model need to solve a dynamic filtering problem in order to make optimal consumption and price setting decisions. In a given period, the weight agents put on new information is inversely related to the precision of their priors. Since a man-bites-dog signals in period t can increase the prior uncertainty in period $t+1$, agents may put more weight on all signals in period $t+1$ relative to the case when there was no man-bites-dog episode in period t. The increased sensitivity to new information can persist for several periods and implies that the impact of an exogenous disturbance of a given magnitude can also be larger than usual for several periods. The mechanism can thus generate periods of higher volatility of macro economic aggregates as observed in the data and documented by Engle (1982), Stock and Watson (2003), Primiceri (2005) and Fernandez-Villaverde and Rubio-Ramirez (2010) among others.

Bloom (2009) and Bloom, Floetotto and Jaimovich (2011) analyze models in which firms respond to increased uncertainty by adopting a wait-and-see approach to capital investment and recruiting so that an increase in the second moment of the exogenous shocks generates a fall in output. The papers by Bloom and Bloom *et al* thus provides a story for how firms respond to increased uncertainty and where uncertainty has a direct effect on the level of output. The man-bites-dog mechanism provides a story for how economic agents come to understand that conditional uncertainty has increased. In the model presented here, a large shock in levels is more likely to generate a man-bites-dog signal and a large shock in levels is thus more likely to lead to an increase in the conditional uncertainty. In related work, Bachmann and Moscarini (2011) propose an alternative mechanism that also implies that causality runs from realizations of first moment shocks to conditional uncertainty. In their paper, imperfectly informed firms tend to be more likely to experiment with prices after a large negative shock. This behavior allows firms to get a better estimate of the demand elasticity of the good that they are producing, but increases the conditional variance of profits in the short run.

As mentioned above, observing a man-bites-dog signal can increase the posterior uncertainty relative to the case when no man-bites-dog signal is observed. The same parameter restrictions that ensure that posterior variances increases after a man-bites-dog signal also imply that the cross-sectional dispersion of expectations increases. This holds even if the signal is public (in the strong common knowledge sense of the word). In the data, we observe a positive correlation between the cross-sectional dispersion of forecasts (as measured by the Survey of Professional Forecaster) and the absolute magnitudes of changes in macro aggregates. Interpreted through the lens of the model, this suggest that the empirically relevant specification of the model may be one where the increase in uncertainty from conditioning on the availability of a man-bites-dog signal is dominating the increased precision due to the content of the signal.

In order to quantify the importance of the man-bites-dog aspect of news reporting I estimate the model on US data. In addition to standard macro variables like GDP, CPI inflation and the Federal Funds rate, I also use the quarterly time series of Total Factor Productivity constructed by John Fernald (2010) as well as individual survey responses from the *Survey of Professional Forecasters*. Using individual survey responses, i.e. the entire cross-section of individual survey responses rather than a mean or median response, has at least two advantages. First, and as documented by Mankiw, Reis and Wolfers (2004) there is significant time variation in the dispersion of forecasts reported by survey respondents. Since the model can potentially fit this fact, raw survey data can be exploited when estimating the model, allowing for a sharper inference about the precision of signals observed by agents. The second advantage of using individual survey responses rather than a median or mean expectation stems from the fact that the number of survey respondents varies over time. For instance, the number of respondents forecasting nominal GDP growth and CPI inflation varies between 9 (1990:Q2) and 50 (2005:Q4) in a sample that covers the period 1981:Q3 to 2010Q4. Using raw survey data and likelihood based estimation methods naturally incorporates that we have a (presumably) more representative sample of the population with 50 observations than with 9.

Most existing macro models imply that the dispersion of cross-sectional expectations is either zero, as in the full information rational expectations models, or non-zero but constant as in models with private but time-invariant information structures, e.g. Lorenzoni (2009), Mackowiak and Wiederholt (2009), Graham and Wright (2009), Nimark (2008, 2010) Angeletos and La'O (2009, 2010) or Melosi (2011). One exception is the sticky information models of Mankiw and Reis (2002) and Reis (2006a, 2006b). In sticky information models, only a fraction of agents update their information in each period and those who update, all observe the state perfectly. Because of this feature, the cross-sectional distribution of expectations implied by sticky information models is a mixture of degenerate distributions, with relative weights decreasing with the vintage of information that the forecasts are based on. It is because the implied cross-sectional dispersion of expectations in the model presented here is time-varying but continuous that it is possible to estimate the structural parameters of the model using likelihood based methods and individual survey responses. One methodological contribution of the paper is to demonstrate how dynamic models with a time-varying information structure can be solved and estimated. This may be of independent interest to some readers.

The paper is structured as follows. In the next section, the concept of man-bites-dog signals is introduced formally. In the static setting of that section, many results can be derived analytically and it is demonstrated that the implications of man-bites-dog signals crucially depend on two parameters (i) the parameter governing how unusual an event has to be to significantly increase the probability of observing a signal (ii) the variance of the noise in the signal. The larger either of these parameters are, the more likely it is that a man-bites-dog signal will increase uncertainty and cross-sectional disagreement. Section 3 demonstrates that a man-bites-dog signal makes the aggregate response to shocks stronger in the beauty contest game of Morris and Shin (2002). Section 4 presents a simple business cycle model similar to that of Lorenzoni (2009), but with a man-bites-dog type of information structure. Section 5 discusses how the model is solved and how the parameters are estimated. Section 6 contains the main empirical results of the paper. Section 7 briefly discusses two alternative specifications of the model and Section 8 concludes.

2. Signals and unusual events

This section introduces the concept of man-bites-dog signals formally and contains the main theoretical results of the paper. We will start by being intentionally nonspecific about distributions and derive some general properties of man-bites-dog information structures that hold under minimal assumptions. A signal y about a latent variable x will be called a man-bites-dog signal if it is more likely to be available when the realization of x is more unusual. When the availability of the signal y depends on the realized value of x, the availability of y by itself carries information about x independently of the particular realized value of the signal y. Bayes' rule then implies that it is *as if* the latent variable x is drawn from a different distribution when the signal y is available compared to when it is not. In particular, conditional on the signal y being available, the relative probability of unusual events increases. This section draws out the implications of this fact for how Bayesian agents update their beliefs in response to man-bites-dog signals.

2.1. Signal availability and conditional distributions.

We will denote the unconditional probability density function of the latent variable of interest x as $p(x)$. An unusual realization of x is thus a realization for which $p(x)$ is small. We are interested in information structures in which the probability of observing the signal y about x is larger for relatively unusual realizations of x. To help distinguish between a particular realization of the signal y and the event that the signal y is available, the indicator variable S is defined to take the value 1 when the signal y about x is available and 0 otherwise. We can then define a man-bites-dog signal as follows.

Definition 1. *The signal y is said to be a man-bites-dog signal if for any two realizations of x denoted x' and x'' such that*

$$p(x') < p(x'') \tag{2.1}$$

the inequality

$$p(S = 1 \mid x') > p(S = 1 \mid x'') \tag{2.2}$$

holds.

The first inequality in the definition simply establishes that x' is a more unusual realization than x''. The second inequality formalizes the notion that a more unusual realization of x is considered more newsworthy than a more common realization. Because the signal y is more likely to be available for some realized values of x than for others, the availability of the signal y is in itself informative about the distribution of x. More specifically, using Bayes' rule, the next proposition shows that conditional on the event that the signal y is available, the probability of more unusual realizations of x increases.

Proposition 1. *The more unusual realization x' is relatively more likely when the signal y is available, i.e.*

$$\frac{p\left(x' \mid S = 1\right)}{p\left(x'' \mid S = 1\right)} > \frac{p\left(x'\right)}{p\left(x''\right)} \tag{2.3}$$

Proof. Dividing Bayes' rule for conditional probabilities

$$p\left(x \mid S = 1\right) p\left(S = 1\right) = p\left(S = 1 \mid x\right) p\left(x\right) \tag{2.4}$$

for x' by the same expression for x'' gives

$$\frac{p\left(x' \mid S = 1\right)}{p\left(x'' \mid S = 1\right)} = \frac{p(S = 1 \mid x')}{p(S = 1 \mid x'')} \frac{p\left(x'\right)}{p\left(x''\right)} \tag{2.5}$$

The proof then follows directly from the fact that the inequality (2.2) in Definition 1 implies that

$$\frac{p(S = 1 \mid x')}{p(S = 1 \mid x'')} > 1 \tag{2.6}$$

□

Proposition 1 states that the relative probability of observing the more unusual realization x' compared to the more common realization x'' is larger conditional on the signal y being available. The availability of a man-bites-dog signal thus implies that probability mass should be redistributed away from unconditionally more likely outcomes towards relatively less likely outcomes. By a completely symmetric argument we have

$$\frac{p\left(x' \mid S = 0\right)}{p\left(x'' \mid S = 0\right)} < \frac{p\left(x'\right)}{p\left(x''\right)} \tag{2.7}$$

so that the absence of a man-bites-dog signal implies that probability mass should be redistributed towards relatively more likely outcomes. Because the availability of the signal y is a discrete event, whether y is available or not thus effectively splits the unconditional distribution $p(x)$ into the two distinct conditional distributions $p\left(x \mid S = 1\right)$ and $p(x \mid S = 0)$.

2.2. **Unimodal symmetric distributions.** While the implications of a man-bites-dog information structure derived above hold for any distribution $p(x)$, from here on we will restrict our attention to unimodal symmetric distributions centered around a zero mean. The probability density function $p(x)$ then takes a small value when the absolute value of x is large since realizations of x further out in the tails of $p(x)$ are more unusual than realizations closer to the mean.

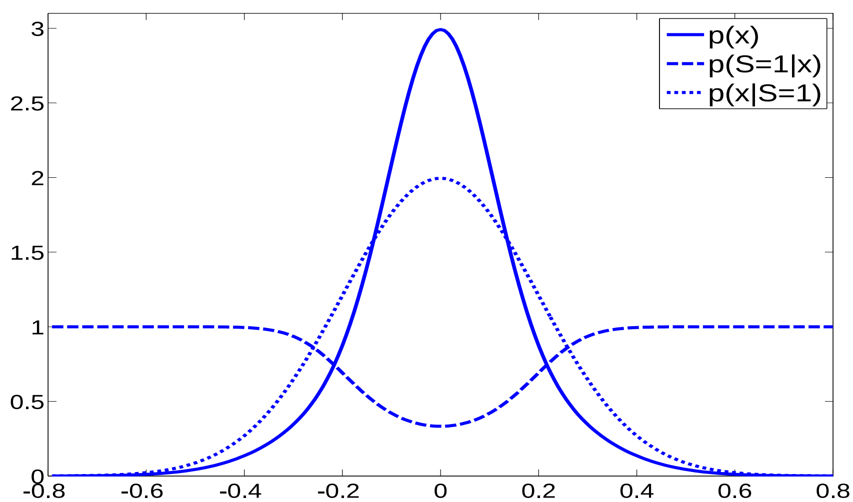

FIGURE 1. Unconditional distribution of x, conditional probability of observing the signal y and the implied conditional distribution of x.

Figure 1 illustrates a man-bites-dog information structure for a unimodal and symmetric distribution $p(x)$ (solid line). The dashed line illustrates the probability of observing the signal y conditional on different realizations of x. At the mean, there is approximately a 40% chance of observing the signal y. As realizations of x further from the mean are considered, the conditional probability of observing a signal increases towards 1 so that a signal y is available almost surely when the realization of x is far enough away from the mean. Graphically, that the conditional probability of observing y satisfies the definition of a man-bites-dog signal is implied by the fact that the slope of the dashed line and the solid line are of opposite signs (or both zero) for all values of x.

The distribution of x conditional on y being available can be backed out from $p(x)$ and the conditional probability of observing y by using Bayes' rule

$$p(x \mid S = 1) = \frac{p(S = 1 \mid x)\, p(x)}{p(S = 1)}. \tag{2.8}$$

if the unconditional probability of observing y is known. By construction, the conditional probability of observing y is increasing in the absolute value of x. Since the conditional distribution on the left hand side of (2.8) is proportional to the product of the unconditional distribution $p(x)$ and the conditional probability $p(S = 1 \mid x)$, the conditional distribution $p(x \mid S = 1)$ must have more probability mass in tails of the distribution relative to the unconditional distribution $p(x)$. This is illustrated by the dotted line in Figure 1.

Bayes' rule can also be used to plot the conditional distribution of x when y is not available. From the inequality (2.7) we know that it must have less probability mass in the tails than $p(x)$ (not shown). A man-bites-dog information structure thus introduces a form of conditional heteroscedasticity in the distribution of x. From the agents' perspective, it is *as if* the latent variable x is drawn from a more dispersed distribution when the signal y is available compared to when it is not. It is important to note that this is true even when the signal content of y is not specifically about the variance, or second moment of, x. It is also important to keep in mind that the indicator variable S is a modeling device that we use to describe the event that the signal y is available and not a separate signal that agents can observe directly and independently of y.

2.3. **Reverse engineering a tractable man-bites-dog information structure.** So far, little has been assumed about the signal y apart from how the probability of observing it depends on the realized value of x. In order to describe a complete filtering problem of an economic agent, we need to be more specific not only about the exact nature of the signal y but also about other potential sources of information.

Throughout the rest of this section as well as in the next, agents indexed by $j \in (0, 1)$ want to form an estimate of x conditional on all available information. There are two types of signals. Agent j can always observe the private signal x_j which is the sum of the true x plus an idiosyncratic noise term

$$x_j = x + \varepsilon_j : \varepsilon_j \sim N\left(0, \sigma_\varepsilon^2\right) \ \forall \ \ j. \tag{2.9}$$

where the variance of the idiosyncratic noise term ε_j is common across agents. There also exists a public signal y

$$y = x + \eta : \eta \sim N\left(0, \sigma_\eta^2\right). \tag{2.10}$$

that may not always be available. As before, the indicator variable S takes the value 1 when y is available and 0 otherwise. The fact that all agents observe y when it is available is common knowledge (though this does not really matter until later).

To make y a man-bites-dog signal we need to specify how the availability of y depends on the realized value of x. One approach would be to directly write down a parameterized functional form for the conditional probability $p(S = 1 \mid x)$ that conforms to the definition of a man-bites-dog signal. That approach is feasible, but will in general not be sufficient to deliver tractable expressions for agents' posterior beliefs. Instead, we will take the following approach.

The agents in the model always know whether the signal y is available or not and they will never need to evaluate the unconditional distribution $p(x)$. When agents observe the signals x_j and y they will thus update their beliefs from the conditional, "prior" distribution $p(x \mid S)$. We can ensure tractability by directly specifying these conditional distributions to be of a class that delivers closed form solutions for agents' posterior beliefs. By applying the results of Proposition 1 "in reverse" when we parameterize these conditional distributions we can ensure that the inequalities in Definition 1 are satisfied so that we indeed have a man-bites-dog information structure.

2.3.1. *A tractable class of conditional distributions.* Updating normally distributed priors to normally distributed signals results in normally distributed posteriors. A natural choice given the signal structure (2.9) - (2.10) is thus to make the distribution $p(x \mid S)$ conditionally normal. To this end, specify x conditional on S as

$$p(x \mid S=0) = N\left(0, \sigma^2\right) \tag{2.11}$$

$$p(x \mid S=1) = N\left(0, \gamma\sigma^2\right) \tag{2.12}$$

so that the unconditional distribution $p(x)$ is a mixture normal

$$x \sim (1-\omega) N\left(0, \sigma^2\right) + \omega N\left(0, \gamma\sigma^2\right) \tag{2.13}$$

The parameter ω then determines how often the signal y is observed in the unconditional sense, i.e. $\omega \equiv p(S=1)$.

2.3.2. *Verifying Definition 1.* The distributional assumptions (2.11) - (2.12) ensures tractability of agents' posterior beliefs. To make y a man-bites-dog signal, we need to parameterize these distributions so that the probability of observing y is larger for values of x that are relatively less likely under the unconditional distribution (2.13). The probability density function of a mixture of two normals both centered at zero is decreasing in the absolute value of x. Larger realizations of x are thus more unusual so for y to be a man-bites-dog signal we need the probability $p(S=1 \mid x)$ to be increasing in the absolute value of x. If we substitute the distributional assumptions above into Bayes' rule we get the expression

$$\frac{p(S=1 \mid x)}{1-p(S=1 \mid x)} = \frac{\omega}{(1-\omega)} \frac{1}{\sqrt{\gamma}} e^{\left(1-\gamma^{-1}\right)\frac{x^2}{2\sigma^2}} \tag{2.14}$$

The term $(1-\gamma^{-1})$ in the exponent on the right hand side of (2.14) that multiplies the square of x is positive if $\gamma > 1$. Imposing this restriction on γ thus ensures that the conditional probability $p(S=1 \mid x)$ of observing the signal y is increasing in the absolute value of x. It is also clear from the expression that setting $\gamma = 1$ makes the probability of observing y independent of x. (The expression (2.14) is derived in the Appendix.)

2.3.3. *Manipulating the probability of observing y.* Choosing ω, γ and σ^2 let us manipulate the shape of the conditional probability of observing the signal y. For instance, a large value for γ implies that the probability of observing the signal y is low for values of x close to its mean. Similarly, observing y can be made a rare event by setting ω close to zero. However, even when it is unusual to observe y in the unconditional sense, it is the parameter γ that determines how informative the availability of y is about the conditional distribution of x. With a low ω and a γ close to 1, even though it is very rare to observe y, doing so does not mean that unusual realizations are significantly more likely. This can be seen from that the ratio of the conditional densities

$$\frac{p(x \mid S=1)}{p(x \mid S=0)} = \frac{1}{\sqrt{\gamma}} e^{\left(1-\gamma^{-1}\right)\frac{x^2}{2\sigma^2}} \tag{2.15}$$

is close to 1 when $\gamma \approx 1$. That is, with the parameter γ approximately equal to unity, observing y is does not imply a conditional distribution of x that is very different from its unconditional distribution.

While manipulating the parameters ω and γ we may want to treat the unconditional variance of x as a primitive that we want to keep fixed. For all values of ω and γ this is always possible since the variance of the mixture normal distribution (2.13) is given by

$$\sigma_x^2 = \omega\gamma\sigma^2 + (1-\omega)\,\sigma^2 \tag{2.16}$$

and thus provides sufficient flexibility to hold σ_x^2 fixed by scaling σ^2.

2.4. **Posterior beliefs.** Given the definitions (2.9) and (2.10) of the signals x_j and y and the distributional assumptions (2.11) - (2.12), agent $j's$ conditional expectation of x are given by standard formulas for multiple signals with independent Gaussian noise processes. Denoting agent j's information set Ω_j^0 when $S=0$ and Ω_j^1 when $S=1$ we have

$$E\left(x \mid \Omega_j^0\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}} x_j \tag{2.17}$$

and

$$E\left(x \mid \Omega_j^1\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} y \tag{2.18}$$

The weights on the signals are determined by the relative precision of the individual signals and the respective conditional distribution. The posterior variances are also given by standard formulas

$$E\left[x - E\left(x \mid \Omega_j^0\right)\right]^2 = \left(\sigma_\varepsilon^{-2}+\sigma^{-2}\right)^{-1} \tag{2.19}$$

or

$$E\left[x - E\left(x \mid \Omega_j^1\right)\right]^2 = \left(\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}\right)^{-1}. \tag{2.20}$$

depending on wether the signal y is available or not.

2.5. **Properties of conditional expectations with *man-bites-dog* signals.** The expressions for the posterior beliefs above allow us to prove two results that may at first appear counterintuitive, but that are arguably natural consequences of Proposition 1. First, the posterior uncertainty can be larger after the signal y has been observed, compared to when it has not been observed. Second, the dispersion of expectations about x may increase after y is observed, even though y is a public signal.

Proposition 2. *The posterior uncertainty about x is larger when the signal y is observed relative to when it is not if the inequality*

$$\sigma_\eta^2 > \frac{\sigma^2}{(1-\gamma^{-1})} \tag{2.21}$$

holds.

Proof. From (2.19) and (2.20) it is clear that the posterior variance is larger when y is available if the inequality

$$\sigma_\varepsilon^{-2}+\sigma^{-2} > \sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2} \tag{2.22}$$

holds. The proof follows by rearranging (2.22) into the inequality in the proposition. □

There are two effects that are active when agents update their beliefs in response to y and they influence agents' posterior uncertainty in opposite directions. By Proposition 1 we know that the information revealed about x by the event that the signal y is *available* make agents redistribute probability mass towards the tails of the distribution. This increases uncertainty. But observing the contents of the signal y is informative about the location of x which decreases uncertainty. Clearly, this second effect will be weaker when the signal is very noisy. When y is sufficiently noisy for the inequality (2.21) to hold, the former effect dominates and the posterior variance is larger than it would be if y was not available.

For a fixed variance of the noise in the signal y, the inequality in the proposition will also hold if γ is sufficiently large. A large γ implies that only very unusual realizations of x are significantly more likely to generate the signal y. The distribution $p(x \mid S = 1)$ then has lot more probability mass in the tails relative to the unconditional distribution $p(x)$ and the signal y do not have to be very noisy for this effect to dominate. However, since the right hand side of (2.21) has a minimum of σ^2 when $\gamma \to \infty$, that $\sigma_\eta^2 > \sigma^2$ is a necessary condition for the uncertainty to be larger when y is available.

Since uncertainty may increase after observing y one may think that risk averse agents would be better off if the signal y was never available. This is not the case. It can be shown that even when the private signal x_j is uninformative, the unconditional expectation of the posterior variance of agents' beliefs is strictly smaller than the unconditional variance σ_x^2 as long as the man-bites-dog signal is not infinitely noisy. This is related to a result from information theory stating that in general, it is possible that some realizations of signals may increase entropy, though on average entropy must decrease when conditioning on more information (see Theorem 2.6.5 of Cover and Thomas 2006).[1]

Corollary 1. *When the inequality*

$$\sigma_\eta^2 > \frac{\sigma^2}{(1 - \gamma^{-1})} \tag{2.23}$$

holds, the cross sectional dispersion of expectations about x is larger when y is observed compared to when it is not.

The proof follows directly from that the denominator in the weight on the private signal is the same as the denominator in the posterior variances. The cross sectional dispersion is increasing in the weight on the private signal, holding the variance of the idiosyncratic noise constant. The same conditions that deliver a higher posterior variance thus also deliver more weight on the private signal and the intuition is also similar. If the public noise variance is high and the conditional probability of a tail event is high, agents will put more weight on other (e.g. private) sources of information.

This result can be contrasted to that of Kondor (2010). He shows that in a setting where two classes of agents are constrained in what type of private information they can acquire, a public signal may increase the dispersion between first and second order expectations. In the model presented here, a man-bites-dog signal decreases dispersion between different orders of expectation (not shown) but increases the cross-sectional dispersion of first order expectations.

[1]I am indebted to Mirko Wiederholt for pointing out this link to me.

It is straight forward to show that the total weight agents put on all signals increases when $S = 1$.

Proposition 3. *The (cross-sectional) average expectation of x responds stronger to x when $S = 1$ than when $S = 0$.*

Proof. In the Appendix. □

The proof of Proposition 3 simply entails verifying that the sum of the coefficients on the two signals is larger when $S = 1$ than the coefficient on the single private signal when $S = 0$. The weights agents put on y and x_j are inversely related to the precision of the conditional distributions $p(x \mid S)$ and the sum of these weights thus go up unambiguously when y is available. As will be shown below, this implies that aggregate responses to shocks are amplified by the presence of a man-bites-dog signal.

3. Man-bites-dog signals in a beauty contest game

Above, the implications of a man-bites-dog information structure for agents' beliefs about the latent variable x were analyzed in some detail, but there were no economic decisions made by the agents. Before turning to the dynamic business cycle model of the next section, it is helpful to first consider the implications of man-bites-dog signals in a static game. Here we introduce the information structure presented above into the beauty contest model of Morris and Shin (2002). This simple model will help us build intuition for the how a man-bites-dog information structure will affect the dynamic business cycle presented in the next section by linking the responses of agents expectation analyzed above to the implied aggregate responses in the presence of strategic complementarities. Below the main components of Morris and Shin's model are presented, though many of the derivations are relegated to the Online Appendix.

3.1. A beauty contest model.

The model of Morris and Shin (2002) consists of a utility function U_j for agent j

$$U_j = -(1-r)(a_j - x)^2 - r\left(L_j - \overline{L}\right) \tag{3.1}$$

where a_j is the action taken by agent j. The first quadratic term in the utility function implies that agent j wants to take an action that is close to the value of the unobserved variable x. The second term introduces a strategic motive since

$$L_j \equiv \int (a_i - a_j)^2 \, di \tag{3.2}$$

and

$$\overline{L} \equiv \int L_i di \tag{3.3}$$

Maximizing the expected value of the utility function (3.1) results in a first order condition for agent j given by

$$a_j = (1-r) E[x \mid \Omega_j] + rE[\overline{a} \mid \Omega_j] \tag{3.4}$$

where $\overline{a}$ is the cross-sectional average action

$$\overline{a} \equiv \int a_i di \tag{3.5}$$

For a positive value of r, agent j thus want to take an action that is close to the true value of x as well as close to the average action taken by other agents. The relative weight of these two objectives is determined by the parameter r. This basic structure is identical to that of the model in Morris and Shin (2002). As shown by Angeletos, Iovino and La'O (2011), agent $j's$ first order condition (3.4) is isomorphic to that of a firm in a simple business cycle model with monopolistic competition and dispersed information. In that setting, the action a_j corresponds to the optimal level of firm $j's$ output and the parameter r is a composite function of the parameters governing the curvature of the utility function and the elasticity of substitution between differentiated goods.

3.2. **The average action as a function of higher order expectations.** The expectation about the average action of other agents can be eliminated from agent $j's$ first order condition by repeated substitution. Taking averages of the resulting expression allows us to rewrite the average action $\overline{a}$ as a weighted average of higher order expectations about x

$$\overline{a} = (1-r)\sum_{k=1}^{\infty} r^{k-1}x^{(k)} \tag{3.6}$$

where the average k order expectation $x^{(k)}$ is defined recursively as

$$x^{(k)} \equiv \int E\left[x^{(k-1)} \mid \Omega_j\right] dj \tag{3.7}$$

starting from the convention that $x^{(0)} \equiv x$. The expression for the average action (3.6) holds regardless of the assumed information structure. A man-bites-dog information structure will thus imply a different average action $\overline{a}$ compared to the original model only to the extent that such an information structure will imply that higher order expectations in the two models differ.

3.3. **Higher order expectations and signals.** We can now embed the information structure defined in Section 2.3 into the model described above. Given the signal structure (2.9) and (2.10) and the distributional assumptions (2.11) - (2.12) the average k order expectation about x is given by

$$x^{(k)} = g_0^k x \tag{3.8}$$

when agents only observe x_j and by

$$x^{(k)} = g_y y + g_x^k (x - g_y y) \tag{3.9}$$

when the signal y is also available. The coefficients g_0, g_x and g_y are given by

$$g_0 \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}}, \quad g_x \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}, \quad g_y \equiv \frac{\sigma_\eta^{-2}}{\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} \tag{3.10}$$

From the expression for the k order expectation (3.9) we can see that

$$\lim_{k\to\infty} x^{(k)} = g_y y \tag{3.11}$$

since $0 < g_x < 1$. Thus, just as in Morris and Shin's model, higher order expectations tend to be dominated by the public signal y (when available) as the order of expectation increases.[2]

3.4. **Aggregate responses to shocks and man-bites-dog signals.** In order to determine how the average action $\overline{a}$ is affected by the man-bites-dog structure, substitute the expressions for the higher order expectations (3.8) and (3.9) into the average action expression (3.6). After simplifying, the average action is given by

$$\overline{a} = \frac{(1-r)\,g_0}{1-rg_0}x \tag{3.12}$$

when the signal y is not available and

$$\overline{a} = \frac{(1-r)\,g_x}{1-rg_x}x + \left(1 - \frac{(1-r)\,g_x}{1-rg_x}\right) g_y y \tag{3.13}$$

when it is. We can then prove the following.

Proposition 4. *The response of the average action $\overline{a}$ to a given value of x is stronger when the signal y is available.*

Proof. In the Appendix. □

Proving the proposition entails verifying that the coefficient on x in (3.12) is always smaller than the sum of the coefficients on x and y in (3.13). The proposition holds for all parameter values, including when $\gamma = 1$ and is thus true partly because expectations (of all orders) simply respond stronger when there are more signals available. Perhaps more interestingly, the next proposition establishes that conditional on the signal y being available, the average action responds stronger to a man-bites-dog signal than to a standard public signal of the same precision.

Proposition 5. *Holding the precision of the signals and the unconditional variance of x fixed, the response of the average action denoted $\overline{a}^*$ is given by*

$$\overline{a}^* = \frac{(1-r)\,g_x^*}{1-rg_x^*}x + \left(1 - \frac{(1-r)\,g_x^*}{1-rg_x^*}\right) g_y^* y \tag{3.14}$$

where

$$g_x^* \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma_x^{-2}}, \quad g_y^* \equiv \frac{\sigma_\eta^{-2}}{\sigma_\eta^{-2} + \sigma_x^{-2}} \tag{3.15}$$

The response of the average action $\overline{a}$ to a given value of x is stronger if y is available and is a man-bites-dog signal compared to the response of $\overline{a}^*$ in the case when the signal y is always available.

Proof. In the Appendix. □

[2]In the original model of Morris and Shin (2002), the unconditional variance of x (or θ in their notation) is left undefined. Yet, the original model is a special case of the set-up here. The equilibrium is identical for the two models if we impose the parameter restrictions $\gamma = \omega = 1$ and by letting σ_x^2 (which then equals σ^2) tend to infinity.

The proof of Proposition 5 uses the following two features of the model. First, the average action in (3.6) is an increasing function of all orders of expectations. Second, all orders of expectations respond stronger to a given x when y is man-bites-dog signal compared to when it is not. To understand the second part, note that when $\gamma > 1$ so that y is a man-bites-dog signal, by (2.16) the conditional variance $var(x \mid S = 1)$ is larger than the unconditional variance, while if y is always available the unconditional and the conditional variance are by construction the same. Since agents are willing to update their first order expectations further when the conditional variance is large, first order expectations will respond stronger to a change in x when y is man-bites-dog signal. This in turn implies that the conditional variance of first order expectations is higher when y is a man-bites-dog signal making agents willing to update their expectations about other agents' expectations further as well. Second order expectations thus also respond more to a given change in x. Applying the same argument to third and higher order expectations explains why all orders of expectations respond stronger to x when y is a man-bites-dog signal.

A man-bites-dog information structure also changes how noise in the signal y affects the average action relative to the case when y is always available. Since $y = x + \eta$ the coefficient on y in (3.13) determines how big an impact a noise shock has on the average action $\overline{a}$. For the same reason that the average action responds stronger to the fundamental x when a man-bites-dog signal is available, the impact of a noise shock in man-bites-dog signal is also stronger compared to a noise shock in a standard signal. Of course, with a man-bites-dog information structure, the noise shock can only affect the average action when the signal y is available. This limits how much of the unconditional variance that can be explained by a noise shock.

4. A business cycle model

This section presents a simple business cycle model, following closely that of Lorenzoni (2009) but with a man-bites-dog information structure. As in the model by Lorenzoni, there are informationally separated islands that are subject to both common and island specific productivity shocks that cannot be distinguished by direct observation. Instead, island inhabitants need to form an estimate of the common component of productivity in order to make optimal consumption and price setting decisions. In the model, there is both island specific and public information and the man-bites-dog signal is specified as a public signal about aggregate productivity. This means that information about large changes to productivity will be more correlated across agents than information about small changes.

4.1. Preferences and technology.

The set up is a stationary version of the island economy described in Lorenzoni (2009). There is a continuum of islands indexed by $j \in (0, 1)$ and on each island there is a continuum of firms indexed by $i \in (0, 1)$ producing differentiated goods. On each island there is a representative household that consume goods and supply labor $N_{j,t}$ to the firms on the island. Households are thus also indexed by $j \in (0, 1)$ and a household on island j maximizes

$$E \sum_{s=0}^{\infty} \beta^s \left[\exp(d_{j,t}) \ln C_{j,t} - \frac{N_{j,t}^{1+\varphi}}{1+\varphi} \mid \Omega_{j,t} \right] \tag{4.1}$$

where $\Omega_{j,t}$ is the information set of inhabitants of island j and $C_{j,t}$ is the consumption bundle consumed by island j households defined as

$$C_{j,t} = \left(\int_{\mathcal{B}j,m} \int_0^1 C_{i,m,j,t}^{(\delta-1)/\delta} di \; dm \right)^{\delta/(1-\delta)} \tag{4.2}$$

As in Lorenzoni (2009), households only consume a subset $\mathcal{B}_{j,m} \subseteq (0,1)$ of the available goods in the economy and the set $\mathcal{B}_{j,m}$ is drawn by nature in each period. The shock $d_{j,t}$

$$d_{j,t} = d_t + \zeta_{j,t} : \zeta_{j,t} \sim N\left(0, \sigma_\zeta^2\right) \tag{4.3}$$

is a demand disturbance that is correlated across islands and the common component d_t follows an AR(1) process

$$d_t = \rho_d d_{t-1} + u_t^d : u_t^d \sim N\left(0, \sigma_d^2\right) \tag{4.4}$$

The demand disturbance d_t is not present in the original model by Lorenzoni (2009) but is needed here in order to avoid stochastic singularity when the model is estimated. Firm i on island j produce good i, j using the technology

$$Y_{i,j,t} = \exp\left(a_{j,t}\right) N_{i,j,t} \tag{4.5}$$

(The log of) productivity $a_{j,t}$ is the sum of a common component a_t and the island specific component $\varepsilon_{j,t}$

$$a_{j,t} = a_t + \varepsilon_{j,t} : \varepsilon_{j,t} \sim N\left(0, \sigma_\varepsilon^2\right) \;\; \forall \;\; j, t. \tag{4.6}$$

The common productivity component a_t follows an AR(1) process

$$a_t = \rho_a a_{t-1} + u_t^a \tag{4.7}$$

The innovation to common productivity u_t^a is distributed as a mixture normal and is specified in detail below. Firms on island j are owned by island j households and set prices $P_{j,t}$ to maximize discounted expected profits $\Pi_{j,t}$

$$E\left[\sum_{s=t}^{\infty} \theta^s \beta^s \frac{C_{j,t}}{C_{j,t+s}} \Pi_{j,t+s} \mid \Omega_{j,t}\right] = E\left[\sum_{s=t}^{\infty} \theta^s \beta^s \frac{C_{j,t}}{C_{j,t+s}} \left(P_{j,t+s} Y_{j,t+s} - W_{j,t+s} N_{j,t+s}\right) \mid \Omega_{j,t}\right] \tag{4.8}$$

where θ is the probability of not changing the price of a given good in a given period. The intertemporal budget constraint of households on island j equates expenditure with income

$$\frac{B_{j,t+1}}{R_t} + \int_{\mathcal{B}j,m} \int P_{i,j,t} C_{j,m} di \; dm = B_{j,t} + W_{j,t} N_{j,t} + \int \Pi_{i,j,t} di \tag{4.9}$$

R_t is the nominal one period interest rate which (in logs) follows a Taylor rule

$$r_t = \phi_\pi \pi_t + \phi_y y_t + \phi_r r_{t-1} + u_t^r : u_t^r \sim N\left(0, \sigma_r^2\right) \tag{4.10}$$

and $B_{j,t}$ are holdings of nominal bonds that pay one dollar in period t by households on island j.

4.2. **Linearized equilibrium conditions.** The model presented above can be log linearized around a non-stochastic steady state, yielding the following equilibrium conditions. (i) An Euler equation determining the optimal intertemporal allocation of consumption

$$c_{j,t} = E\left[c_{j,t+1} \mid \Omega_{j,t}\right] - r_t + E\left[\overline{\pi}_{\mathcal{B}j,t+1} \mid \Omega_{j,t}\right] + d_{j,t} \tag{4.11}$$

where $\overline{\pi}_{\mathcal{B}j,t+1}$ is the inflation of the goods basket consumed on island j in period $t+1$. (ii) A labor supply condition equating marginal disutility of labor supply with the marginal utility of consumption multiplied by the real wage

$$w_{j,t} - \overline{p}_{\mathcal{B}j,t} = c_{j,t} + \varphi n_{j,t}. \tag{4.12}$$

(iii) A demand schedule for good j depending on the relevant relative price of good j

$$y_{j,t} = \int_{\mathcal{C},j,t} c_{m,t} dm - \delta \left(p_{j,t} - \int_{\mathcal{C},j,t} \overline{p}_{m,t} dm \right) \tag{4.13}$$

where $\int_{\mathcal{C},j,t} \overline{p}_{m,t} dl$ is the log of the relevant price sub index for consumers from other islands buying goods from island j. (iv) An island j Phillips curve relating inflation on island j to the nominal marginal cost on island j and expected future island j inflation

$$\begin{aligned} p_{j,t} - p_{j,t-1} &= \lambda \left(\overline{p}_{\mathcal{B}j,t} + c_{j,t} - p_{j,t} - a_{j,t} \right) + \lambda\varphi \left(y_{j,t} - a_{j,t} \right) \\ &\quad + \beta E \left(p_{j,t+1} - p_{j,t} \mid \Omega_{j,t} \right) \end{aligned} \tag{4.14}$$

where $\overline{p}_{\mathcal{B}j,t}$ is the relevant price subindex for consumers on island j and $\lambda = (1-\theta)(1-\theta\beta)/\beta$. The steps required to arrive at the linearized equilibrium conditions (4.11) - (4.14) are identical to those described in Lorenzoni (2009).

4.3. **Local information.** The inhabitants of each island observe their own productivity and demand disturbances $a_{j,t}$ and $d_{j,t}$. Since these contain a component that is common across islands, these local variables are informative about the aggregate state. In addition to these exogenous local signals, firms and households can also extract information about the aggregate state from observing the demand for their own good $y_{j,t}$ and the price of the goods basket that they purchase. Lorenzoni (2009) assumes that the islands visited by the inhabitant of island j while shopping are drawn so that the price index of the goods basket consumed by island j inhabitants is equal to the aggregate price level plus a normally distributed island j specific shock. We will make a similar assumption, with an adjustment to the mean of the normally distributed shock such that the signal is conditionally stationary. That is, the set $\mathcal{B}_{j,t}$ is drawn such that

$$\overline{p}_{\mathcal{B}j,t} = p_t + \xi^p_{j,t} : \xi^p_{j,t} \sim N(p_{j,t-1} - p_{t-1}, \sigma^2_{\xi p}) \tag{4.15}$$

Since $p_{j,t-1}$ is observable by the inhabitants of island j the price index of the good purchased by island j inhabitant is thus a noisy measure of aggregate inflation $p_t - p_{t-1}$, rather than of the aggregate price level as in Lorenzoni (2009). Reformulating the signal structure this way does not change anything substantial in the model but simplifies the representation of agents' filtering problems since all other variables are stationary (while the price level is not). Similarly, we assume that the set of islands $\mathcal{C}_{j,t}$ are drawn such that (4.13) takes the form

$$y_{j,t} = y_t - \delta \left(p_{j,t} - p_t \right) + \xi^y_{j,t} : \xi^y_{j,t} \sim N(\delta \left[p_{j,t-1} - p_{t-1} \right], \sigma^2_{\xi y}) \tag{4.16}$$

Again, the adjustment of the mean of the disturbance relative to Lorenzoni (2009) is made in order to keep signals stationary. As in Lorenzoni's original model, the shocks $\xi_{j,t}^{p}$ and $\xi_{j,t}^{y}$ are introduced in order to prevent local interactions from perfectly revealing the aggregate state.

4.4. **The joint distribution of signals and shocks.** The man-bites-dog signal structure is embedded in the business cycle model in a similar way as in the static setting discussed in Section 2 and 3. The unobservable variable of interest is the common component of productivity a_t. The joint distribution of the indicator variable s_t and the innovations u_t^a in (4.7) are specified such that a man-bites-dog signal is more likely to be generated when there has been a large (in absolute terms) innovation to the common productivity process a_t. The indicator variable s_t takes the value 1 when a man-bites-dog signal is generated in period t which occurs with unconditional probability ω. Similar to the static setting of Section 2, a mixture normal density for u_t^a will be used to keep the filtering problem tractable

$$u_t^a \sim (1-\omega) N\left(0, \sigma_a^2\right) + \omega N\left(0, \gamma\sigma_a^2\right). \tag{4.17}$$

with $\gamma > 1$. The unconditional variance of the productivity innovations u_t^a is then given by

$$E\left(u_t^a\right)^2 = (1-\omega)\,\sigma_a^2 + \omega\gamma\sigma_a^2 \tag{4.18}$$

To complete the description of the joint distribution of innovations and signals, it is further assumed that when $s_t = 1$ all households observe an additional public signal $z_{a,t}$ given by

$$z_{a,t} = a_t + \eta_t : \eta_t \sim N\left(0, \sigma_\eta^2\right) \tag{4.19}$$

The signal z_t^a is thus a man-bites-dog signal and the vector of observables $\mathbf{z}_{j,t}$ available to households and firms on island j in period t then is

$$\mathbf{z}_{j,t} = \begin{bmatrix} a_{j,t} & d_{j,t} & y_{j,t} & \overline{p}_{j,t} & r_t & s_t \end{bmatrix}' \tag{4.20}$$

if $s_t = 0$ and

$$\mathbf{z}_{j,t} = \begin{bmatrix} a_{j,t} & d_{j,t} & y_{j,t} & \overline{p}_{j,t} & r_t & s_t & z_t^a \end{bmatrix}' \tag{4.21}$$

if $s_t = 1$. The information set $\Omega_{j,t}$ of firms and households on island j evolves as

$$\Omega_{j,t} = \left\{\mathbf{z}_{j,t}, \Omega_{j,t-1}\right\} \tag{4.22}$$

This completes the description of the model. It is perhaps worth noting that the original model of Lorenzoni (2009) is nested in the model presented here by setting $\rho_a = \omega = \gamma = 1$ and $\sigma_d^2 = 0$.

5. Solving and estimating the model

There are two features of the model presented above that make standard solution methods for linear rational expectations models inapplicable. First, there is island specific information about variables of common interest to all islands. Natural state representation then tend to become infinite, due to the well-known problem of the infinite regress of expectations that arises when agents need to "forecast the forecasts of others" (see Townsend 1983 and Sargent 1991). Second, the precision of agents' information is a function of the realized history of s_t and thus varies over time. This section briefly outlines how the solution method proposed in Nimark (2011) can be modified to solve a model with a time-varying information structure. Details are kept at a minimum and focus is on aspects of the solution method that help intuition for how time-varying information sets translate into time-varying equilibrium dynamics. A complete description of the solution algorithm is provided in the Online Appendix. When solved, the model is in standard state space form, albeit with time-varying parameters. The section ends with a description of how a posterior estimate of the parameters of the model and the history of s_t can be constructed using the Multiple-Block Metropolis-Hastings algorithm described in Chib (2001).

5.1. **Rationality and the dynamics of higher order expectations.** Nimark (2011) proposes an approximation method to solve linear rational expectations models with privately informed agents. Conceptually, the solution method has two components. The first is to put structure on higher order expectations, i.e. expectations about other agents' expectations, by exploiting that it is common knowledge that agents form model consistent expectations. The second part is to use that the impact of higher order expectations on inflation and output is decreasing in the order of expectation. The second part is somewhat involved, and interested readers are referred to the original reference for more details. Here we briefly describe how common knowledge of model consistent expectations helps put structure on the dynamics of higher order expectations.

Let $\mathbf{x}_t$ denote a vector containing the exogenous state variables a_t and d_t so that

$$\mathbf{x}_t \equiv \begin{bmatrix} a_t \\ d_t \end{bmatrix}. \tag{5.1}$$

With island specific information, the state of the model need to be expanded relative to the full information case to also include average higher order expectations of current productivity a_t and the common demand shock d_t. The state can then be represented by the vector X_t defined as

$$X_t \equiv \begin{bmatrix} \mathbf{x}_t' & \mathbf{x}_t'^{(1)} & \cdots & \mathbf{x}_t'^{(\overline{k})} \end{bmatrix}' \tag{5.2}$$

where

$$\mathbf{x}_t^{(k+1)} \equiv \int E\left[\mathbf{x}_t^{(k)} \mid \Omega_t(j)\right] dj. \tag{5.3}$$

The constant $\overline{k}$ is the maximum order of expectation considered.

To solve the model we need to find the law of motion for the vector X_t. The law of motion of $\mathbf{x}_t$, i.e. the first component of X_t, is given by (4.4) and (4.7). As usual in rational expectations models, first order expectations $\mathbf{x}_t^{(1)}$ are optimal, i.e. model consistent estimates of the actual exogenous state vector $\mathbf{x}_t$. The knowledge that other traders form model consistent estimates allow traders to treat average first order expectations as a stochastic process with known properties when they form second order expectations. Common knowledge of the model thus implies that second order expectations $\mathbf{x}_t^{(2)}$ are optimal estimates of $\mathbf{x}_t^{(1)}$ given the law of motion for $\mathbf{x}_t^{(1)}$. Imposing this structure on all orders of expectations allows us to find the law of motion for the hierarchy of expectations X_t as a function of the structural parameters of the model.

The agents inhabiting the islands of the model use the Kalman filter to form an estimate of all the average higher order expectations in the state vector X_t. Island j's estimate of X_t

follows the update equation

$$\underbrace{E\left[X_t \mid \Omega_{j,t}\right]}_{posterior} = (I - K_t D_t)\underbrace{E\left[X_t \mid \Omega_{j,t-1}\right]}_{prior} + K_t \underbrace{\mathbf{z}_{j,t}}_{signal} \tag{5.4}$$

where K_t is the Kalman gain and D_t is a matrix that maps the state into the observable vector $\mathbf{z}_{j,t}$. The Kalman filter thus plays a dual role. It is used by individual agents to form an estimate of the state vector. But since the aggregate state vector X_t is made up of the cross-sectional average of individual state estimates determined by (5.4), the Kalman update equation above also determines the law of motion of the aggregate state X_t.

5.2. **Time-varying state dynamics.** The Kalman update equation (5.4) determines how agents combine prior beliefs with period t signals. When signals are very precise or when the prior is very uncertain, agents will put more weight on the signals and less weight on the prior. For the same reasons that expectations respond stronger in the static example of Section 2, agents' expectations about productivity will respond stronger to a productivity innovation of a given size when a man-bites-dog signal is available.

A man-bites-dog event may also have persistent effects on the relative weight on the prior and the signals in agents' update equation. The reason is that if, for example, there is a man-bites-dog episode in period t that increases the posterior uncertainty in period t, then this will translate into an increase in prior uncertainty in period $t+1$. A larger posterior uncertainty in period t thus translates into more weight being put on signals observed in period $t+1$. Through this channel, the economy may become more responsive to structural shocks for several periods after a man-bites-dog event.

Since a period t realization of the indicator variable s_t may have persistent effects on the dynamics of the state X_t we need to keep track of the history of s_t (which we denote s^t) to determine period t equilibrium dynamics. However, as long as the filtering errors of agents follow a stable process, i.e. do not accumulate over time, we do not need to keep track of the entire history of s_t but only its most recent realizations. How far back in time the realizations of s_t matter depends on the eigenvalues of the process that propagates the variance of agents' filtering errors through time. If the underlying processes are not very persistent, or if information is very accurate, filtering errors do not tend to be long lived and only a few of the past realizations of s_t influence current dynamics. In general, how many lags of s_t that are relevant for period t dynamics depend on the parameters of the model and needs to be checked on a case-by-case basis.

The strategy we follow is to specify a maximum lag of s_t, say $s_{t-\mathcal{T}}$, where $\mathcal{T}$ should be large enough so that the Kalman gain K_t in agents' update equation (5.4) is invariant to changes in the histories of s_t up to period $t-\mathcal{T}$. That is, setting $\mathcal{T}=4$ is an appropriate choice if the changes in the Kalman gain K_t depending on whether $s_{t-\mathcal{T}-1}, s_{t-\mathcal{T}-2}$, ...etc equals 1 or 0 are so small that it does not justify the increased computational burden of including one more lag of s_t. With 2 different exogenous regimes (i.e. $s_t \in \{0,1\}$) there will be $2^{\mathcal{T}}$ relevant different histories s^t and $2^{\mathcal{T}}$ different Kalman gains K_t in the update equation (5.4).[3]

[3]It is perhaps worth pointing out here that while we truncate the history of s_t used to compute the Kalman gain K_t, agents still condition on the entire history of observables $\mathbf{z}_{j,t}$.

The equilibrium law of motion of X_t is a vector autoregression of the form

$$X_t = M(s^t)X_{t-1} + N(s^t)\mathbf{u}_t : \mathbf{u}_t \sim N(0, I) \tag{5.5}$$

where the dependence of the matrices M and N on s^t is a consequence of the dependence of the law of motion of the state on the the Kalman gain K_t in (5.4). There are thus also $2^{\mathcal{T}}$ different laws of motion, or endogenous regimes if you like, for X_t.

5.3. **Aggregate inflation and output.** Taking averages of the consumption Euler equation (4.11) and the Phillips curve (4.14) and collecting the resulting expressions in vector form gives the vector Euler equation

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = A \int E\left(\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \end{bmatrix} \mid \Omega_{j,t} \right) + (B + C)\, X_t + G_r r_{t-1} \tag{5.6}$$

linking current inflation and output to the average expectation of the same variables in the next period. To solve out the expectations term we conjecture (and verify) that inflation and output can be expressed as linear functions of the state X_t with period t parameters depending on the history of man-bites-dog events s^t

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = G(s^t)X_t + G_u\mathbf{u}_t + G_r r_{t-1} \tag{5.7}$$

The matrices G_u and G_r captures the direct effect of the monetary policy shock and lagged interest rate respectively.[4] Since the interest rate is observable these matrices do not depend on s^t. When forming expectations about period $t+1$ inflation and output agents will need to take into account the probability that there will be a man-bites-dog signal available in the next period. Expectations of output and inflation thus depend on the probability ω that s_{t+1} will take the value 1. The time varying matrix $G(s^t)$ can be computed by noting that for a given conjectured law of motion (5.5) and the linear function (5.7) we can express current inflation and output as

$$\begin{aligned} \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} &= \omega A G\left(s_1^{t+1}\right) M\left(s_1^{t+1}\right) H X_t \\ &\quad + (1-\omega)\, A G\left(s_0^{t+1}\right) M\left(s_0^{t+1}\right) H X_t \\ &\quad + (B + C)\, X_t + G_u\mathbf{u}_t + G_r r_{t-1} \end{aligned} \tag{5.8}$$

where s_n^{t+1} denotes the history s^{t+1} with $s_{t+1} = n$. The matrix H is defined so that

$$\int E\left[X_t \mid \Omega_{j,t}\right] dj = HX_t \tag{5.9}$$

That is, H moves a vector of average higher order expectations one step "up" in orders of expectations and is used in (5.8) to compute the average expectation in (5.6). Equating

[4]There is also an indirect affect of a monetary policy shock on inflation and output since such a shock will affect the higher order expectations in X_t. This effect is time-varying since it works through the matrices $N(s^t)$ and $G(s^t)$.

coefficients in (5.7) and (5.8) implies

$$\begin{aligned} G\left(s^{t}\right) &= \omega AG\left(s_{1}^{t+1}\right)M\left(s_{1}^{t+1}\right)H \\ &\quad + (1-\omega)AG\left(s_{0}^{t+1}\right)M\left(s_{0}^{t+1}\right)H \\ &\quad + (B+C) \end{aligned} \tag{5.10}$$

The matrices $G(s^t)$ depend on the law of motion (5.5) through $M(s^{t+1})$ in (5.10). At the same time, the matrices $M(s^t)$ and $N(s^t)$ in the law of motion for the state X_t depends on $G(s^t)$. This dependence arise because how informative the signal vector $\mathbf{z}_{t,j}$ is about the state X_t is partly a function of $G(s^t)$. The Online Appendix describes a fixed point algorithm that can be used to find the equilibrium dynamics of the model for each of the $2^{\mathcal{T}}$ relevant histories of s_t.

5.4. **Estimating the model.** The solved model is a state space system with time-varying parameters and standard likelihood based methods are applicable to estimate the parameters of the model. In addition to the 19 structural parameters of the benchmark model, which we denote

$$\Theta = \{\rho_a, \rho_d, \sigma_a, \sigma_d, \sigma_r, \sigma_\varepsilon, \sigma_\zeta, \sigma_{\xi 1}, \sigma_{\xi 2}, \sigma_\eta, \delta, \eta, \phi_\pi, \phi_y, \phi_r, \theta, \beta, \omega, \gamma\} \tag{5.11}$$

we also want to construct a posterior estimate of the indicator variable s_t that keeps track of whether there was a man-bites-dog signal available in period t or not. Below we describe how this can be done by sampling from the two conditional distributions $p\left(\Theta \mid s^T, Z^T\right)$ and $p\left(s^T \mid \Theta, Z^T\right)$. Dividing the sampling from the joint posterior distribution of Θ and s^T into two conditional blocks lets us get around the problem that unlike the agents inside the model, as econometricians we do not observe the regimes s_t directly but conditional on a given history s^T, the model is linear-Gaussian and it is straightforward to evaluate the likelihood.

5.5. **The data.** The time series used to estimate the model are US CPI inflation, the Federal Funds rate, US real GDP, the quarterly time series of total factor productivity constructed by John Fernald (2010) and individual survey response data from the *Survey of Professional Forecasters* (SPF). The data is quarterly and the sample ranges from 1981:Q3 to 2010:Q4. The start date is chosen based on the availability of survey data for inflation forecasts and the end date is the date of the most recent data on real GDP and total factor productivity. CPI inflation is de-trended using a linear trend and the same trend is taken out of the Federal Funds rate. Real GDP is de-trended using the HP-filter with a smoothing coefficient of 1600. The survey data used are the individual survey responses of one quarter ahead forecasts of CPI inflation and nominal GDP growth taken from the *Survey of Professional Forecasters* available from the web site of the Federal Reserve Bank of Philadelphia. Inflation forecasts are de-trended using the CPI inflation trend. Nominal GDP growth forecasts are de-trended by subtracting the inflation trend and the growth in the real GDP (HP-filter) trend.

We denote the vector of observables in period t as Z_t. All elements in Z_t have natural counterparts in the model. Specifically, TFP will be taken as a noisy measure of the common productivity component and the vector of survey forecasts $\mathbf{f}_t'$ are taken to be representative of the expectations of the inhabitants of the islands in the model.

In the benchmark specification, the vector of observables are thus given by

$$Z_t = \left[\begin{array}{ccccc} a_t & \pi_t & y_t & r_t & \mathbf{f}_t' \end{array}\right]' \tag{5.12}$$

and linked to the state of the model by the measurement equation

$$Z_t = \overline{D}(s^t)X_t + D_r r_{t-1} + \overline{R}(s^t)\mathbf{u}_t \tag{5.13}$$

Due to the fact that the number of survey respondents is not constant in the sample, the dimensions of both $\overline{D}$ and $\overline{R}$ are time-varying. More interestingly, the entries of $\overline{D}$ and $\overline{R}$ also varies over time. The matrix $\overline{D}$ is time-varying since the function mapping the state into endogenous variables is time-varying. The matrix $\overline{R}$ is time-varying since the cross-sectional dispersion of forecasts are time-varying in the model and each survey entry can be viewed as a noisy measure of the average expectation where the variance of the "noise" is the cross-sectional variance of forecasts implied by the model. Individual respondents can be tracked in the SPF so while it is possible to exploit the (limited) panel dimension of the SPF responses in theory, it is not feasible in practise. Doing so would require that in order to evaluate the likelihood, we as econometricians would need to carry along an individual state for each respondent in the SPF, thereby increasing the state dimension by a multiplier of 50. Instead, we treat individual survey responses of inflation and nominal GDP growth forecasts as independent draws from the distributions

$$f_{t,\pi}^j \sim N\left(\int E\left[\pi_{t+1} \mid \Omega_{j,t}\right] dj, \sigma^2_{f\pi}(s^t)\right) \tag{5.14}$$

and

$$f_{t,\pi+\Delta y}^j \sim N\left(\int E\left[\Delta y_{t+1} + \pi_{t+1} \mid \Omega_{j,t}\right] dj, \sigma^2_{f\pi+\Delta y}(s^t)\right) \tag{5.15}$$

respectively, where the variances $\sigma^2_{f\pi}(s^t)$ and $\sigma^2_{f\pi+\Delta y}(s^t)$ are the model implied cross-sectional variance of inflation and nominal GDP growth expectations. The cross-sectional variances are a function of the structural parameters Θ as well as the history s^t and thus vary over time.

5.6. **Priors.** We will use uniform priors on most of the parameters in Θ. The exceptions are the parameter on inflation in the Taylor rule ϕ_π and the parameter ρ_a governing the persistence of productivity. In preliminary estimations, these parameters tended to regions where the model becomes nearly unstable, i.e. ϕ_π tended to 1 from above and ρ_a tended to 1 from below. This is due to the fact that generating instability is an "easy" way for the model to generate the forecast dispersion observed in the individual survey responses. Since the number of survey responses used is large relative to the number of standard macro economic time series, whether a given parametrization fits the dispersion in the survey data or not has a large impact on the likelihood function. It is also clear that the tendencies of ϕ_π and ρ_a to approach 1 are not driven by the joint empirical properties of interest rates and inflation or the persistence of the TFP series since this problem disappears when the model is estimated using only the mean of surveys rather than the full cross-section of survey responses. Imposing priors that ensures that the parameters do not approach instability regions is thus a way to ensure that the estimation algorithm does not get stuck in a local maxima where it fits the dispersion of forecasts well, but at the expense of fitting the other variables.

5.7. **Estimation Algorithm.** The posterior distribution of Θ and s^T are estimated using a Multiple-Block Metropolis algorithm (see Chib 2001). It exploits that conditional on a history of man-bites-dog regimes s^T, the model is in linear-Gaussian state space form. An outline of the algorithm is as follows:

(1) Specify initial values Θ_0 and s_0^T.
(2) Repeat for $j = 1, 2, ..., J$
    (a) Block 1: Draw Θ_j from $p\left(\Theta \mid s_{j-1}^T, Z^T\right)$
    (b) Block 2: Draw s_j^T from $p\left(s^T \mid \Theta_j, Z^T\right)$
(3) Return values $\{\Theta_0, \Theta_1,...,\Theta_J\}$ and $\{s_0^T, s_1^T,...,s_J^T\}$

The Multiple-block Metropolis algorithm is similar to (and nests as a special case) the Gibbs sampling algorithm. In both algorithms, the parameters are divided into blocks to exploit that it is simpler to sample from the conditional distributions $p\left(\Theta \mid s_{j-1}^T, Z^T\right)$ and $p\left(s^T \mid \Theta_j, Z^T\right)$ than from the joint posterior $p\left(s^T, \Theta \mid Z^T\right)$. However, here, both Step 2(a) and 2(b) are executed using a Metropolis step rather than by drawing directly from the full conditional distribution. The Online Appendix describes the algorithm in detail.

Table 1

Posterior Parameter Estimates 1981:Q3-2010:Q3

		Benchmark		Inflation Survey Response		NGDP Survey Responses		Mean of Surveys	
Θ	Prior Dist	Mode $\widehat{\Theta}$	2.5%-97.5%	Mode $\widehat{\Theta}$	2.5%-97.5%	Mode $\widehat{\Theta}$	2.5%-97.5%	Mode $\widehat{\Theta}$	2.5%-97.5%
Preferences etc									
φ	$U(0,10)$	2.62	2.28-2.82	0.91	0.86-1.19	0.02	0.0078-1.49	1.11	1.06-1.17
δ	$U(0,10)$	0.17	0.08-0.37	0.77	0.77-0.94	0.47	0.40-0.98	0.91	0.87-0.97
β	$U(0.96,1)$	0.99	0.99-0.99	0.99	0.99-0.99	0.99	0.98-0.99	0.98	0.97-0.99
θ	$U(0,1)$	0.81	0.79-0.83	0.84	0.83-0.87	0.87	0.85-0.90	0.85	0.84-0.87
Exogenous aggregate processes									
ρ_a	$U(0,0.99)$	0.98	0.97-0.98	0.96	0.95-0.97	0.96	0.92-0.97	0.89	0.87-0.93
ρ_d	$U(0,0.99)$	0.35	0.33-0.38	0.83	0.80-0.89	0.71	0.51-0.73	0.79	0.74-0.86
σ_a	$U(0,0.99)$	0.0052	0.0048-0.0053	0.0056	0.0049-0.006	0.0063	0.0052-0.0070	0.0032	0.0031-0.0048
σ_d	$U(0,1)$	0.035	0.031-0.037	0.0093	0.0074-0.010	0.014	0.014-0.018	0.0049	0.0039-0.0055
σ_r	$U(0,1)$	0.0077	0.0069-0.0086	0.006	0.0053-0.0067	0.0037	0.0031-0.0060	0.0049	0.0044-0.0055
Island specific processes									
σ_ε	$U(0,1)$	0.098	0.081-0.11	0.078	0.045-0.094	0.18	0.047-0.20	0.10	0.093-0.11
$\sigma_{\xi p}$	$U(0,1)$	0.0068	0.0063-0.0072	0.010	0.0089-0.011	0.065	0.019-0.099	0.045	0.018-0.063
$\sigma_{\xi y},$	$U(0,1)$	0.18	0.15-0.20	0.090	0.082-0.13	0.76	0.32-0.76	0.010	0.0075-0.016
$\sigma_\zeta,$	$U(0,1)$	0.26	0.21-0.28	0.047	0.016-0.065	0.91	0.29-0.92	0.027	0.0061-0.038
Man-bites-dog parameters									
ω	$U(0,1)$	0.20	0.19-0.28	0.41	0.30-0.43	0.46	0.17-0.48	0.23	0.21-0.26
γ	$U(1,10)$	4.43	4.12-4.60	4.71	4.52-5.01	2.77	2.40-5.13	4.80	4.40-4.87
σ_η	$U(0,10)$	0.18	0.16-0.20	0.22	0.20-0.25	0.040	0.028-0.047	0.084	0.060-0.12
Taylor rule parameters									
ϕ_r	$U(0,1)$	0.023	0.001-0.038	0.16	0.17-0.22	0.41	0.05-0.44	0.19	0.17-0.22
ϕ_π	$U(1,10)$	1.46	1.37-1.52	1.20	1.16-1.28	1.50	1.24-1.58	1.35	1.28-1.42
ϕ_y	$U(0,10)$	0.31	0.29-0.34	0.71	0.64-0.75	0.10	0.086-0.14	0.24	0.19-0.29
Measurement error variance in survey means									
$\sigma^2_{\overline{f\pi}}$	$U(0,1)$	-	-					0.0015	0.0012-0.0016
$\sigma^2_{\overline{f\pi+\Delta y}}$	$U(0,1)$	-	-					0.0081	0.0070-0.0090

5.8. **Posterior parameter estimates.** The posterior mode and the 95 per cent probability intervals for the parameters in Θ are reported in the column labeled *Benchmark* in table 1. The results are based on 2 000 000 draws from the Multiple-block Metropolis algorithm and the Online Appendix contains diagnostic checks for convergence and plots of the posteriors distributions. Most parameters appear to be well-identified. By themselves, most of the parameters are of no particular interest. The exceptions are the parameters that more directly relate to the man-bites-dog information structure, i.e. γ, ω and σ_η.

The unconditional frequency of man-bites-dog events are estimated to be around 20 per cent, with the probability interval reaching. How this interact with γ to determine the probability of observing man-bites-dog signal conditional on the magnitude of the innovation to productivity is analyzed in the next section. The standard deviation of the noise in the man-bites-dog signal is large relative to the precision of the island specific productivity. Nevertheless, and as demonstrated in the next section, agents attach sufficient weight on the man-bites-dog signal so that a pure noise shock has a substantial effect on inflation and output.

6. Estimated Man-bites-dog Dynamics

The estimated model can be used to quantify the contribution of the man-bites-dog mechanism to business cycle dynamics. This section describes how the propagation of shocks changes when there is a man-bites-dog signal available and presents estimates of how the probability of observing a man-bites-dog signal depends on the absolute size of the innovation to productivity. In the model, whether a man-bites-dog signal was available or not is encoded by the indicator variable s_t and here we relate the posterior probabilities that $s_t = 1$ to a *News Heard Index* from the Michigan Survey. This section also presents empirical evidence suggesting persistent effects on the sensitivity of output to productivity shocks after a man-bites-dog event.

6.1. **Impulse propagation with and without man-bites-dog signals.** Figure 2 illustrates the impulse responses of inflation (left column) and output (right column) to innovations to productivity (top row), demand shocks (middle row) and noise shock in the man-bites-dog signal (bottom row). The dashed black lines describe the responses when there is no man-bites-dog signal available in any period and the solid blue lines describe the responses when there is a man-bites-dog signal available in the impact period but none before or after. The dotted lines are the 95 per cent credible intervals.

6.1.1. *Productivity shocks.* Inflation falls and output increases in a gradual, hump-shaped pattern after a positive innovation to productivity whether a man-bites-dog signal is available or not. The responses of inflation and output to a productivity shock of a given magnitude is substantially larger if it coincides with the availability of a man-bites-dog signal. The difference is particularly large for the response of output. At the posterior median, the response of output in the impact period is about twice as large when there is man-bites-dog signal available compared to when there is no such signal. The peak response, at about 0.4 of a percentage point compared to 0.25 of a percentage point, is also substantially larger when there is a man-bites-dog signal available. The fall in inflation is about 20 per cent

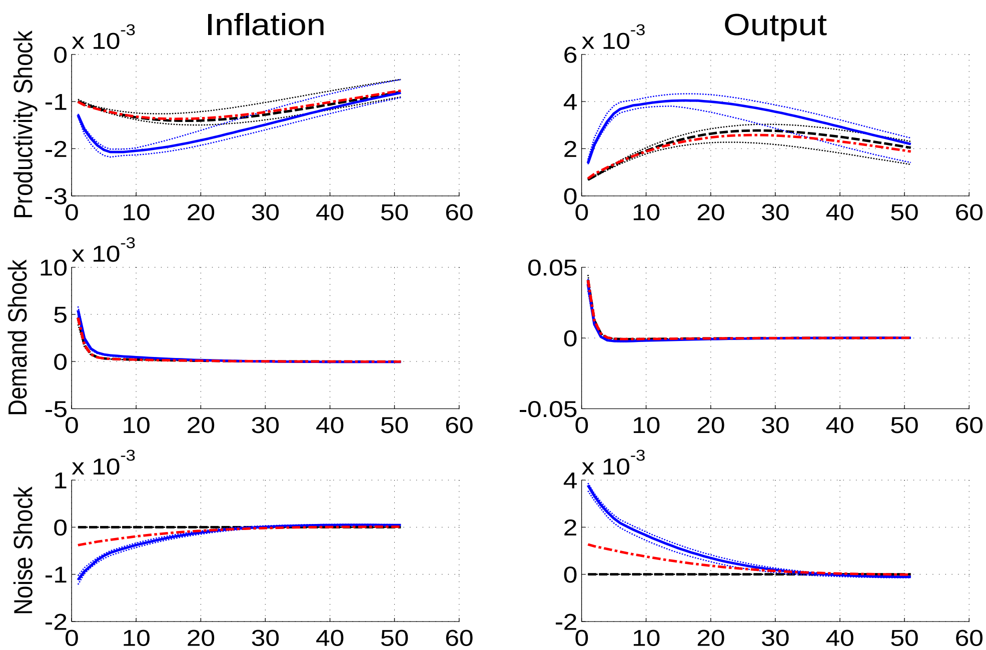

FIGURE 2. Posterior impulse response functions. Blue solid lines are for man-bites-dog signal available in impact period and black dashed lines for no man-bites-dog signal. Dotted lines show the 95 per cent posterior credible intervals. The dash-dot red lines are the responses of inflation and output when the availability of the signal z_t^a is uncorrelated with the innovation to productivity.

larger on impact when a man-bites-dog signal is available. Since the exogenous shock is the same whether a man-bites-dog signal is available or not, the stronger responses are caused by expectations responding stronger when a man-bites-dog signal is available.

In the static model of Section 3 above, the average action responds stronger when a man-bites-dog signal is available partly because expectations simply respond more when there is more information available. However, in Section 3 it was also shown that aggregate responses are stronger to a man-bites-dog signal compared to the response to a standard public signal of the same precision. We can quantify the relative importance of the man-bites-dog effect and the *more information* effect by solving the model under the alternative assumption that the availability of a man-bites-dog signal is uncorrelated with the magnitude of the innovation to productivity. The result of this exercise is illustrated by the red dashed-dotted lines in Figure 2. There we can see that almost the entire difference between the responses with and without a man-bites-dog signal is explained by the man-bites-dog effect.

6.1.2. *Demand shocks.* The middle row of Figure 2 shows that both inflation and output increases after a demand shock. Neither of the responses depend substantially on whether there is a man-bites-dog signal available or not. Both inflation and output respond with a geometric decay after impact, with approximately the same persistence and shape as that of the exogenous shock d_t. This is because relative to the variance of the innovations to d_t, agents have quite precise information about d_t so that (higher order) expectations of demand shocks are close to the actual shock.

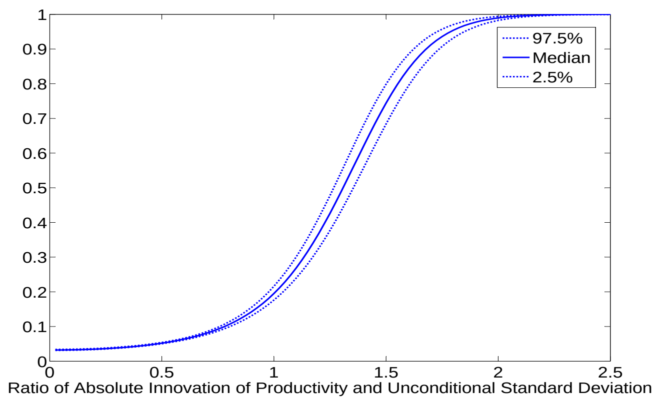

FIGURE 3. The probability of observing a man-bites-dog signal conditional on the absolute size of an innovation to productivity.

6.1.3. *Noise shocks.* The agents in the economy use all available information optimally. Nevertheless, since the man-bites-dog signal is noisy, agents will sometimes inadvertently respond to a pure noise shock. The consequences of this for inflation and output are plotted in the bottom row of Figure 2. Qualitatively, the initial responses of inflation and output to a noise shock in the man-bites-dog signal are similar to the responses to a true innovation to productivity. That is, inflation falls and and output increases. The initial response of both inflation and output is larger to a 1 s.d. pure noise shock than to a 1 s.d. actual productivity shock. This is partly because the s.d. of the noise shock is much larger than the standard deviation of a the true innovations to productivity and for shocks of the same magnitude, the response is weaker to a pure noise shock.

Comparing the response to a noise shock in a man-bites-dog signal to the response when the availability of the public productivity signal is uncorrelated with the underlying shock, about half of the response to a noise shock is due to the *more information* effect. Of course, when there is no signal available, noise shock cannot affect neither inflation nor output, which explains why the black dashed lines in the bottom row are flat at zero.

In the original model of Lorenzoni (2009), the responses to shocks in the public productivity signal look like the responses to demand shocks discussed above (hence the title of Lorenzoni's paper). That is, both inflation and output increases in response to a noise shock. The differences in predictions by the current model and the model of Lorenzoni are not driven directly by the man-bites-dog information structure. For instance, the model presented here also predicts that noise shocks are inflationary if the Taylor rule coefficient on output, i.e. ϕ_y, is sufficiently low (holding the other parameters fixed at their estimated posterior modes). That the posterior estimates do not suggest inflationary noise shocks may be due to the fact that the specification presented here includes "actual" demand shocks, i.e. the same type of shocks Lorenzoni (2009) seeks to replace with noise shocks. By construction, they will absorb much of the positive co-movement between inflation and output.

6.2. **The conditional probability of observing a man-bites-dog signal.** The probability of generating a man-bites-dog signal is increasing in the absolute size of an innovation to productivity when $\gamma > 1$. The larger the innovation is, the more likely it thus is that the response to the shock will be described by the blue impulse responses above rather than the black.

The estimated probability that an innovation to productivity is associated with a man-bites-dog signal conditional on the size of the innovation is illustrated in Figure 3. There, the posterior probability that $s_t = 1$ (y-axis) is plotted against the ratio (x-axis) of the absolute value of an innovation and the unconditional standard deviation of innovations.

We can see in the figure that shocks that are larger than 2 standard deviation is estimated to generate a man-bites-dog signal and the stronger responses of inflation and output almost surely. A man-bites-dog information structure thus introduces a non-linearity in the endogenous variables' responses to exogenous shocks making both recessions and booms sharper than they otherwise would be.

6.3. **Historical man-bites-dog episodes.** The top panel of Figure 4 displays the posterior probabilities of man-bites-dog events for the benchmark specification. The shaded areas are the NBER dated recessions. In addition to the NBER dated recessions, there are several episodes particularly in the 1980s and 2000s that are assigned a high probability of being man-bites-dog episodes. In comparison, the 1990s have much fewer periods with a substantial probability of having been man-bites-dog events.

As suggested by the theory, the quarters that are assigned a high probability of being a man-bites-dog episode are also quarters with larger than average (in absolute terms) productivity innovations. The standard deviation of the innovations in the quarters assigned a probability larger than 50 per cent is 0.0072 compared to 0.0062 for the full sample. The corresponding numbers for quarters with a probability of a man-bites-dog event above 90 per cent is 0.0075. To ensure that the this result is not an artefact of the structure imposed on the data by the model, the innovations were computed from a separate univariate AR(1) model for the TFP series. (The numbers are very similar when the innovations from the full model are used, though the TFP process is then estimated to be more persistent compared to the estimates from the separate TFP model.)

6.4. **News media and historical man-bites-dog episodes.** In the introduction, the *Movers* segment on Bloomberg Television was given as an example of man-bites-dog news reporting. It seems unlikely that individual events, perhaps with the exception of the 1987 stock market crash, directly cause what is identified as man-bites-dog episodes by the model. An alternative interpretation of these episodes at the macro level is that at certain times the economy for various reasons become one of the main news stories, dominating network news and newspaper front pages. According to the man-bites-dog dictum, this should be more likely to happen when macro economic developments are in some sense unusual.

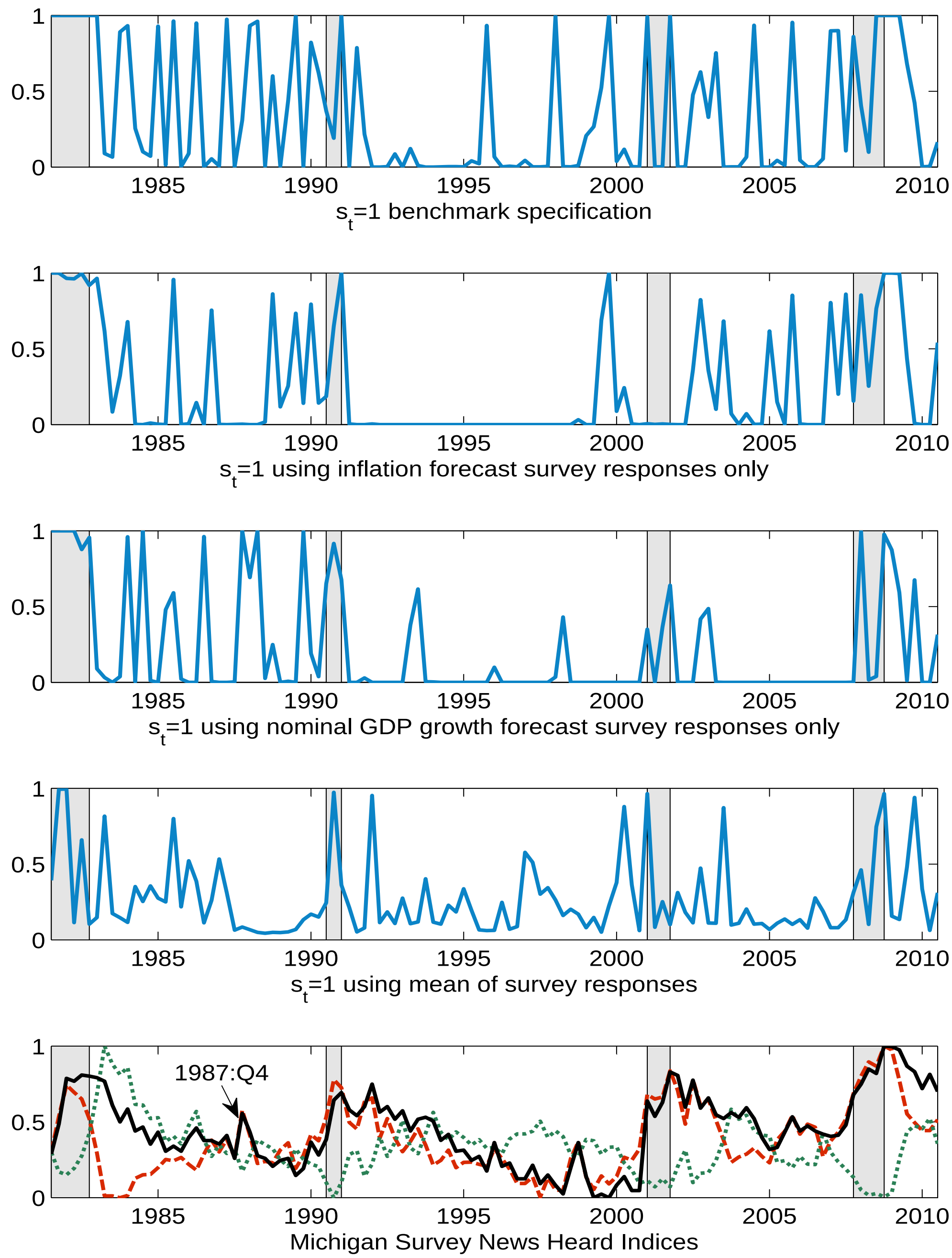

FIGURE 4. Posterior historical probability of $s_t = 1$ and Michigan Survey News Heard indices.

One way to check more directly whether what the model interprets as man-bites-dog events are indeed related to the intensity of news coverage is to compare the posterior probabilities that $s_t = 1$ with the fraction of respondents in the Michigan Survey that have heard either unfavorable or favorable news "during the last months". This data was not used in estimation and thus provides an independent check on how reasonable the estimates produced by the model are. The bottom panel of Figure 4 contains an index of the number of respondents that have heard any news about the economy (black solid line), any unfavorable news about the economy (red dashed line) or any favorable news about the economy (green dotted line).[5] It is clear that the unfavorable news heard index increases around recessions and around the stock market crash in Q4 of 1987. The biggest spike in the good news index is in the early 1980s as the economy recovered from the Volcker disinflation recession.

In addition to these large and easily identifiable events there is a lot of high-frequency variation in the indices. To analyze more formally how the *Any News Heard Index* relates to what the model interprets as man-bites-dog episodes, we can compute the posterior correlation between s_t and the index. For the benchmark specification this correlation is 0.30. While the positive correlation between the timing of man-bites-dog events and the *Any News Heard Index* does not provide a direct test of the causal link proposed in this paper, it does provide evidence that what the model identifies as man-bites-dog episodes are indeed periods associated with an increase in the news coverage on the economy.

6.5. **Man-bites-dog events and the cross-sectional dispersion of expectations.** Because of the man-bites-dog information structure, the precision of agents information sets vary over time. This means that the weight agents put on island specific signals, and as a consequence, the cross-sectional dispersion of expectations will vary over time. Since the cross-sectional dispersion of expectations in period t is a function of the history s^t, the time variation in the cross-sectional dispersion of survey responses are particularly important for identifying man-bites-dog episodes in the sample. This is illustrated in Figure 5, where the cross-sectional variance of survey responses are plotted together with the corresponding fitted dispersion from the benchmark specification.

Overall, the model does a good job at fitting both the average level and the time-variation of the dispersion in the inflation survey responses. Two exceptions are the periods with very high dispersion in the early 1980s and the current crisis during which the model under-predicts the dispersion in the data. The model also somewhat over-predicts the inflation forecast dispersion following the stock market crash in 1987:Q4.

The model has a little more difficulty matching the average level and time-variation in the nominal GDP growth forecast dispersion. In particular, the model over-predicts dispersion in normal times and under-predicts the dispersion during the Volcker recession and in Q4 in 1987 when the stock market crashed.

[5]The indices are constructed by computing the fraction of survey respondents that have heard either favorable or unfavorable news. The fractions are re-normalized to have a minimum of 0 and and a maximum of 1 to make them more easily comparable in a single graph. This normalization is an affine transformation and do not affect the correlations computed below.

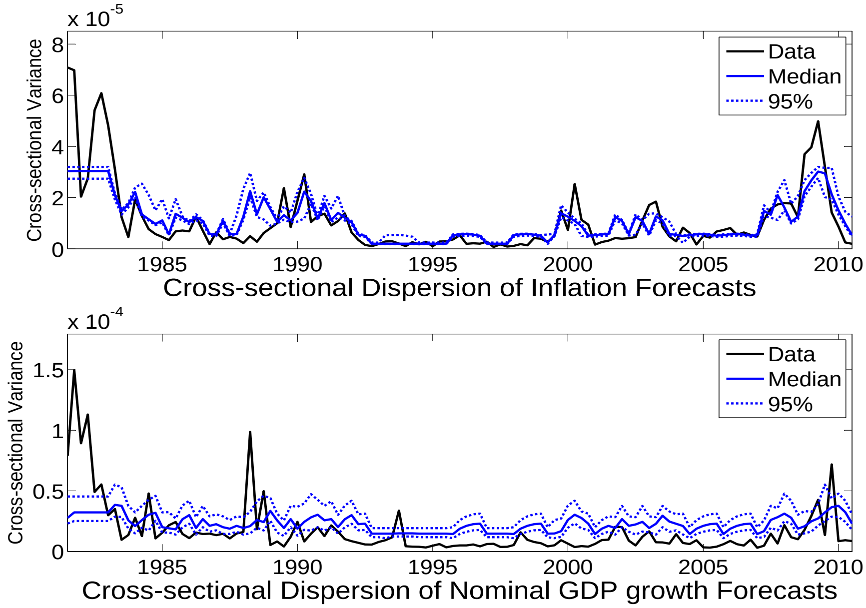

FIGURE 5. Historical and posterior model implied dispersion of survey responses for benchmark specification.

Comparing Figure 4 and 5, it is clear that most of the episodes that the model assigns a high probability of being a man-bites-dog episode is associated with an increase in the dispersion of either the inflation forecasts or the nominal GDP growth forecasts. In particular, the dispersion of survey forecasts are relatively flat in the 1990s and this is also the period with fewer periods with a high probability of being man-bites-dog events.

6.5.1. *Excluding nominal GDP growth survey responses in estimation.* The relationship between the dispersion of survey forecasts and the posterior probabilities of man-bites-dog events is even more apparent if we estimate the model using only one type of survey responses. Here we first describe the implications of excluding the nominal GDP growth forecasts survey responses. (The posterior estimates of the model parameters for this as well as the additional specifications discussed below are are reported in Table 1.) Comparing the two top panels in Figure 4 shows that relative to the benchmark specification, there are fewer quarters that are assigned a high probability of being a man-bites-dog event and they are more clustered in the sample. There is a long period covering most of the 1990s where there are no quarters that are assigned a substantial probability of having been a man-bites-dog event. Also, the 1987 stock market crash and the post-dot-com-boom recession are conspicuously not identified as man-bites-dog events when only the inflation survey responses are used for estimation.

Figure 6 shows that the model is able to fit the inflation forecast dispersion almost perfectly when the nominal GDP growth survey responses are excluded in estimation. This trade-off between fitting the dispersion of the two types of survey forecasts arises because in the

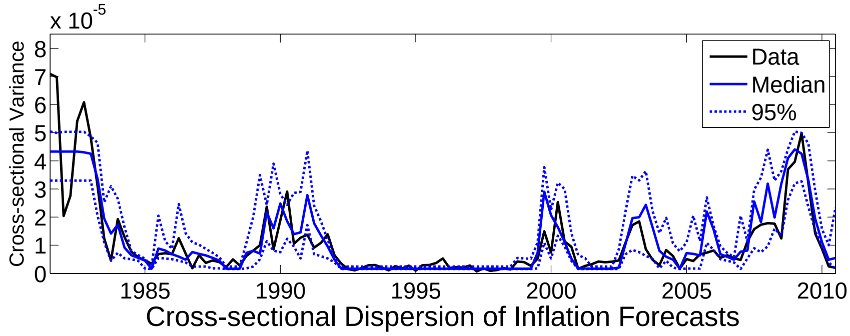

FIGURE 6. Historical and posterior model implied dispersion of survey responses

model, dispersion is a function of a single process, i.e. the history of s_t. In the data, there are episodes when nominal GDP growth forecast dispersion increases while inflation forecast dispersion stays flat. One such episode is 1987:Q4 when the stock market crash caused a large increase in nominal GDP growth forecast dispersion. When only one type of survey data is used for estimation, there is no such trade-off.

Comparing the time series of dispersion in Figure 6 and the posterior estimate of s_t in Figure 5 shows that here is an almost perfect correlation between episodes with probability of $s_t = 1$ and an increase in inflation forecast dispersion in the survey data. Relative to the benchmark specification, the posterior correlation between s_t and the *Any News Heard Index* falls somewhat to 0.28 for this specification.

6.5.2. *Excluding inflation survey responses in estimation.* The middle panel of Figure 5 displays the posterior estimate of s_t when the inflation survey responses was excluded when estimating the model. As with the inflation survey results, there are fewer episodes that are assigned a high probability of being a man-bites-dog event. There is considerable overlap between the s_t series from the inflations forecasts specification, but there are also some differences. For instance, both the 1987 stock market crash and the post-dot-com-boom recession are assigned a high probability of being man-bites-dog events when the nominal GDP growth survey responses are used. The posterior correlation between s_t and the *Any News Heard Index* is similar to that of the benchmark specification at 0.30.

Figure 7 demonstrates that the model is also able to fit the both the average level and time-variation in nominal GDP forecasts dispersion very well when it does not at the same time have to fit the inflation forecast dispersion. Again, there is a close connection between episodes that have a high probability for $s_t = 1$ and an increase in dispersion.

6.5.3. *Using the mean of survey responses.* It is clear from the evidence presented above that the dispersion of survey forecasts are very influential in identifying historical man-bites-dog episodes. It may therefore be of interst to estimate the model without using the information in the cross-section of survey responses. The third panel in Figure 4 shows the posterior estimate for s_t when the mean of the inflation and nominal GDP growth survey responses are used to estimate the model. Relative to the other specifications, there are fewer quarters

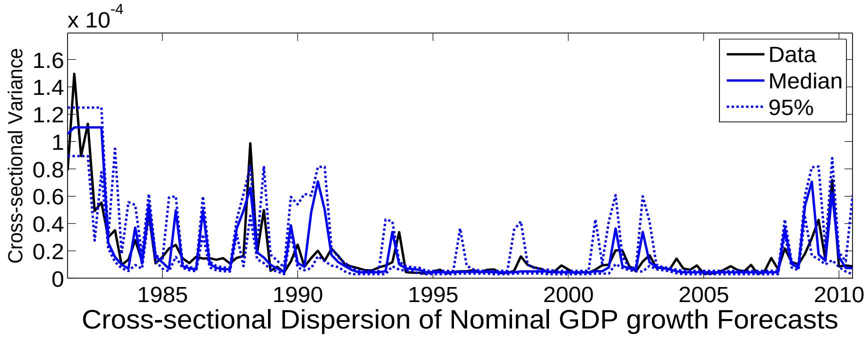

FIGURE 7. Historical and posterior model implied dispersion of survey responses

that are assigned a probability close to 1 of being a man-bites-dog event and there are practically none where a man-bites-dog event is completely ruled out. That the posterior is less precise is not surprising since less sample information was used in estimation.

Without individual survey responses, the model has to identify man-bites-dog episodes from the movements of the aggregate variables and the mean of the surveys. Larger movement of these variables are then required to provide strong evidence of a man-bites-dog event. The standard deviation of the innovations in the quarters assigned a probability larger than 50 per cent is 0.0119, compared to 0.0072 for the benchmark specification. For quarters with a probability of a man-bites-dog event above 90 per cent the standard deviation is 0.0144 compared to 0.075 for the benchmark specification.

Relative to the specifications using individual survey responses, the posterior correlation between s_t and the *Any News Heard Index* drops somewhat to 0.25. That the Index is more strongly correlated with the posterior estiamte of s_t when the individual survey responses are used in estimation provides further support for the model's prediction that news media coverage increases dispersion.

6.6. **The cyclicality of dispersion and uncertainty.** Bloom (2009) reports that the cross-sectional dispersion in the GDP forecasts in the Survey of Professional Forecasts are strongly correlated with measures of uncertainty, for instance stock market volatility measures. Also in the model, uncertainty about the state is positively correlated with increased dispersion in expectations.

Bloom (2009) also documents that uncertainty and cross-sectional dispersion tend to increases around recessions. The correlation between growth rates of nominal GDP and the spread between the 25th and the 75th percentile of nominal GDP growth survey responses is -0.06 over the sample period used in this paper. The model presented here is symmetric and large positive and large negative shocks are equally likely to trigger a man-bites-dog event. There is some support in the data that the magnitude of shocks matter at least as much for dispersion as their sign. While nominal GDP forecast dispersion is negatively correlated with nominal GDP growth rates, the correlation between *absolute* growth rates of nominal GDP and dispersion is 0.31. Absolute changes of the CPI are also more strongly correlated

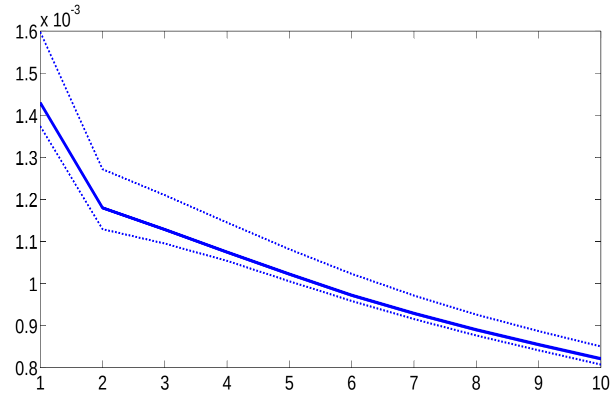

FIGURE 8. Impact multiplier persistence: Impact on output of 1 s.d. innovation to productivity after single man-bites-dog event.

with the dispersion than actual changes. The correlation with the spread is 0.32 for actual changes in CPI and 0.37 for absolute changes.

6.7. **Endogenous persistence in volatility.** A man-bites-dog event is associated with stronger responses of inflation and output to productivity shocks and the impulse response functions in the top panel Figure 2 trace out the responses of inflation and output to an single productivity shock, i.e. when there are no further impulses to productivity after the impact period. However, a single man-bites-dog signal may affect how responsive the economy is to innovations to productivity for several periods after the signal was observed. As explained in Section 5.2 above, if the posterior uncertainty about the state increases in period t then this translates into an increased prior uncertainty in period $t + 1$. Since how much weight agents put on new information depends inversely on the precision of their prior, changes in posterior uncertainty in period t will affect how responsive the economy is to shocks also in period $t + 1$. Figure 8 plots the output multiplier on a productivity shock the periods after a man-bites-dog signal is observed. That is, Figure 7 plots the posterior estimate of the relevant elements of $G(s^t)N(s^t)$ from the model solution (5.5) - (5.7) for $s_t = 1$ in the impact period and $s_t = 0$ before and after.

The impact multiplier is largest in the period when the man-bites-dog signal is observed. A 1 s.d. innovation to productivity increases output by about 0.14 of a percentage point when accompanied by a man-bites-dog signal. The impact multiplier then slowly converges towards 0.008 which is the level associated with no man-bites-dog signals. The man-bites-dog information structure thus generates something similar to ARCH dynamics or stochastic volatility in the endogenous variables even though no such persistence is present in the volatility of the exogenous productivity process.

The effect on the volatility of inflation and output accumulates over time if there are several man-bites-episodes occurring in close succession. This is illustrated in Figure 9 where the posterior estimate of the impact on output of a 1 s.d. innovation to productivity is plotted. In the 1980s when, according to the model, it is likely that there were many

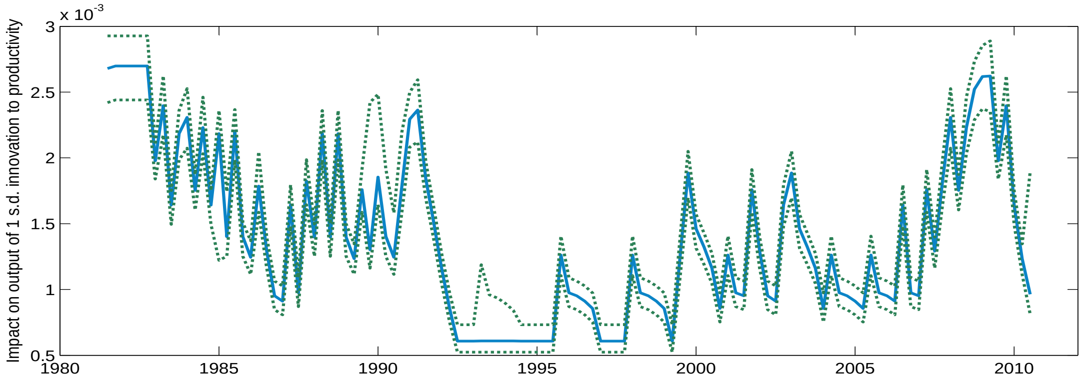

Figure 9. Posterior estimate of historical impact on output of a 1 s.d. innovation to productivity. Median and 2.5%-97.5% probability interval.

man-bites-dog episodes, the impact multiplier is persistently above the level associated with no man-bites-dog events. Only in the more tranquil 1990s do the impact multiplier fall to the level associated with no recent man-bites-dog events. At the peaks in the early 1980s and during the recent financial crisis, the impact multiplier is about 3 times as large as it was in the mid 1990s.

These results are related to the findings of Coibion and Gorodnichenko (2011). Using survey data, they document that expectations are updated faster, in the sense that average expectations in surveys respond stronger, during periods of higher macroeconomic volatility. Though Coibion and Gorodnichenko are looking at a partly different sample period and focus on lower frequency movements, this is exactly the qualitative prediction made by the model presented here: A man-bites-dog event increases the volatility of macro aggregates *because* expectations are updated faster, since a man-bites-dog signal makes agents put more weight on new information.

It is perhaps worth pointing out that the gradual and monotonic decay of the impact multiplier after a man-bites-dog event is an empirical result and not a necessary implication of the model. With a very precise man-bites-dog signal, posterior uncertainty in period t decreases when a man-bites-dog signal is available making agents less sensitive to new information in period $t+1$. The impact multiplier in period $t+1$ would then be lower than that associated with no man-bites-dog signal and would we have observed a negative "overshooting" of the impact multiplier in the periods after the man-bites-dog event occurred.

7. Alternative specifications

In this section I investigate the implications of relaxing some of the assumption implied by the man-bites-dog information structure. In particular, I re-estimate the model under the following three alternative specifications. (i) The public signal z_t^a about common productivity is always available, implying the restriction $\gamma = \omega = 1$. (ii) The signal z_t^a is never available but the innovations to productivity are drawn from the mixture normal distribution

(4.17) with the regime observable directly by the agents. This specification is equivalent to imposing the restriction on the benchmark model that the noise in the man-bites-dog signal has infinite variance, i.e. $\sigma_\eta^2 = \infty$. (iii) The signal z_t^a is always available and the innovations to productivity are drawn from the mixture normal distribution (4.17). The log-likelihood evaluated at the posterior mode of the three specifications as well as for the benchmark specification are listed in Table 2 below.

A simple way to compare the relative fit of different models that takes into account the number of estimated parameters is the Schwarz approximation (see Canova 2007). This approximation to the posterior odds ratio is given by

$$PO \approx e^{\log L\left(\mathbf{y}^T|\widehat{s}_B^T,\widehat{\Theta}_B\right)-\log L\left(\mathbf{y}^T|\widehat{s}^T,\widehat{\Theta}\right)-\frac{1}{2}(\dim\Theta_B-\dim\Theta)\ln T} \tag{7.1}$$

where a B subscript denotes the benchmark specification and $\dim\Theta$ denotes the number of freely estimated parameters. The resulting relative probability of the benchmark model compared to the alternatives are reported in the bottom row of Table 2.

There appear to be overwhelming evidence against the two more restrictive specifications (i) and (ii) which both are assigned a near zero posterior odds ratio relative to the benchmark model. Of these two, allowing for the signal z_t^a to always be available but with a single volatility regime as in (i) appears to fit the data better than specification (ii) that allows for i.i.d. volatility regimes but do not include public signals about productivity.

Specification (iii) is not a nested special case of the benchmark specifications. However, both (iii) and the benchmark specification can be nested within a more general model resulting in the same number of freely estimated parameters.

Table 2

	Benchmark	(i) $\gamma = \omega = 1$	(ii) $\sigma_\eta^2 = \infty$	(iii) Pub. signal + regimes
$\log L\left(Z^T \mid \widehat{s}^T, \widehat{\Theta}\right)$	3087.2	2992.6	2982.7	3089.5
PO	e^0	$e^{89.8}$	$e^{102.1}$	$e^{-2.3}$

According to the posterior odds ratio, the specification allowing for the signal z_t^a to always be available but with time-varying volatility of productivity shocks is about $e^{2.3} \approx 10$ times more likely to have generated the data than the benchmark specification. The increased flexibility of the latter to fit the data is due to that the noise in the public signal provides an additional aggregate shock that in specification (iii) is available in every period.

8. Conclusion

That some types of signals are more likely to be available about unusual events may suggest that we should be better informed about unusual events. However, in this paper we have showed that the flip-side of this argument is that the distribution of events conditional on there being a man-bites-dog signal available is different from the unconditional distribution in ways that may undo the effect of having more information. It was demonstrated that conditioning only on the *availability* of a man-bites-dog signal increases uncertainty, while observing the actual contents of the signal decreases uncertainty. If the likelihood of observing the signal increases substantially only after very unusual events or if the signal is sufficiently noisy, the former effect may dominate so that the posterior uncertainty after observing a man-bites-dog signal is larger than it would be if no such signal was available. Even though the man-bites-dog signal is public, the cross-sectional dispersion of (first order) expectations may increase after it is observed. We also shoed that in the beauty contest model of Morris and Shin (2002), the average action responds stronger to a man-bites-dog signal compared to a standard signal of the same precision.

In the second part of the paper, a simple business cycle model was presented in which large innovations to productivity are more likely to generate a public signal. the estimated model suggest that there have been episodes in recent US history in which the impact of an innovation to productivity on aggregate output was more than three times larger than at other times. The increased sensitivity of macro aggregates to productivity innovations were found to be persistent, lasting about 2 years after a single man-bites-dog event. We also presented corroborative and independent evidence from the *Michigan Survey* that the episodes identified by the model as man-bites-dog events were indeed associated with higher than normal news coverage of the economy. This correlation is stronger when individual survey responses are used which provides further support for the mechanism in the model.

Some features of the model appear to capture a "crisis mentality" in which there is an intense media focus on the economy and yet, while there is more information produced and broadcast about the economy, uncertainty and sensitivity to new information appear to increase. The strongest evidence of man-bites-dog events are indeed found around recessions, but overall the number of man-bites-dog events associated with negative and positive innovations to productivity are about the same and in the model, large positive innovations are as likely as large negative innovations to trigger a man-bites-dog signal. Still, it seems plausible that recessions are different from booms in a more fundamental sense than what is captured here. Having said this, there is nothing apart from tractability concerns that prevents us from embedding the man-bites-dog mechanism in a richer model that could capture this asymmetry.

The model was estimated by likelihood based methods using both the quarterly total factor productivity time series constructed by Fernald (2010) and individual survey responses from the *Survey of Professional Forecasters* along with more standard macro indicators. Using a time series of TFP as an observable variable has obvious advantages in terms of disciplining the model, especially since one of the aims of the paper has been to quantify the extent of time variation in the impact of TFP shocks on other variables. Using the cross-section of individual survey responses allowed us to incorporate the information in the time variation in the dispersion of survey responses into the posterior estimates of the parameters of the model. Particularly, we showed that the cross-sectional dimension in the *Survey of Professional Forecasters* is informative about the the timing of man-bites-dog events. In order to exploit the time variation in the cross-sectional dispersion of the survey data, it is necessary to have a model that can fit this fact. The paper makes a methodological contribution by demonstrating how a model with time-varying information sets can be solved and estimated. This may be of separate interest to some readers.

Conceptually, the information structure proposed here differs from the ex ante perspective taken by most of the existing literature on rational inattention, e.g. Sims (1998, 2003) and Mackowiak and Wiederholt (2009). In that literature, agents pay more attention to those variables that are most useful *on average*. In contrast, here realizations of shocks matter for what type of signals that are available. There is nothing inherent in the rational inattention approach though that makes an ex ante perspective necessary. For instance, Matejka (2011) develops a model of rational inattention in which it is optimal for agents to let the precision of signals depend on the realization of shocks. The information structure in that paper thus also depend on the realizations of shocks. However, the availability of signals in Matejka's model is constant and thus do not carry any additional information about the distribution of the variables of interest.

One limitation of the framework presented here is that the availability of signals depends only on the realized value of exogenous shocks. In practise, unusual developments of endogenous variables are surely also considered newsworthy. However, modeling the availability of man-bites-dog signals as depending on the realized values of endogenous variables is at the moment computationally unfeasible.

The model presented here features a restricted form of stochastic volatility. In less restricted stochastic volatility models such as those of Justiano and Primiceri (2008) and Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez and Uribe (2011), persistence in the volatility of endogenous variables is caused by persistence in the volatility of the exogenous shocks. In the model presented here, the volatility of exogenous productivity is restricted to be an i.i.d. process but the filtering problem of the agents generates persistence in the volatility of the endogenous variables. To the extent that we can observe the exogenous shocks directly, this distinction is a testable difference between the two approaches. Finally, even though the model is conditionally linear, changes in variances have first order effects on the propagation of shocks through the filtering problem of the agents. This aspect of the model does not depend on the man-bites-dog mechanism per se. The solution method proposed here could thus relatively easily be extended to more general stochastic volatility specifications including specifications that allow for persistence in the exogenous regimes.

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Appendix A. Proof of Proposition 3

Proposition 3. *The average expectation of x responds stronger to x when S=1 than when S=0.*

Proof. We need to show that the sum of the coefficients on the private signal x_j and the public signal y in the conditional expectation

$$E\left(x \mid \Omega_j^1\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x_j + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} y \tag{A.1}$$

when $S = 1$ is larger than the coefficient on the private signal

$$E\left(x \mid \Omega_j^0\right) = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}} x_j \tag{A.2}$$

when $S = 0$. Simply comparing the expected average expectation conditional on x for $S = 0$

$$\int E\left[x \mid \Omega_j^0\right] dx = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}} x_j dj \tag{A.3}$$

$$= \left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) x \tag{A.4}$$

and $S = 1$

$$E\left[\int E\left(x \mid \Omega_j^1\right) dx \mid x\right] = \int \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x_j dj \tag{A.5}$$

$$+ \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} x \tag{A.6}$$

$$= \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) x \tag{A.7}$$

means that the proposition is true if the inequality

$$\left(1 - \frac{\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma^{-2}}\right) < \left(1 - \frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}}\right) \tag{A.8}$$

holds. The last expression can with a little algebra be rearranged to

$$\gamma^{-1} < 1 + \sigma_\eta^{-2} \tag{A.9}$$

which is always true since $\gamma > 1$ and $\sigma_\eta^{-2} > 0$. □

Appendix B. Deriving Expression (2.14) in Section 2.X

Start by dividing Bayes rule for conditional probabilities for $p(S=1 \mid x)$

$$p(S=1 \mid x) = \frac{p(x \mid S=1)p(S=1)}{p(x)} \tag{B.1}$$

with the corresponding expression for $p(S=0 \mid x)$

$$p(S=0 \mid x) = \frac{p(x \mid S=0)p(S=0)}{p(x)} \tag{B.2}$$

to get

$$\frac{p(S=1 \mid x)}{p(S=0 \mid x)} = \frac{p(x \mid S=1)}{p(x \mid S=0)}\frac{p(S=1)}{p(S=0)} \tag{B.3}$$

Substitute in the distributional assumptions for $p(x \mid S=1)$ and the unconditional probabilities of observing y

$$\frac{p(S=1 \mid x)}{p(S=0 \mid x)} = \frac{\frac{1}{\sqrt{\gamma}\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\frac{x^2}{\gamma\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\frac{x^2}{\sigma^2}}}\frac{\omega}{1-\omega} \tag{B.4}$$

Rearrange and simplify to get (2.14) from the main text

$$\frac{p(S=1 \mid x)}{1-p(S=1 \mid x)} = \frac{\omega}{1-\omega}\frac{1}{\sqrt{\gamma}}e^{\left(1-\frac{1}{\gamma}\right)\frac{x^2}{2\sigma^2}} \tag{B.5}$$

where we also used that $p(S=0 \mid x) = 1 - p(S=1 \mid x)$.

Appendix C. Proof of Proposition 4

Proposition 4 *The response of the average action $\overline{a}$ to a given value of x is stronger when the signal y is available.*

Proof. We need to prove that

$$\frac{(1-r)\,g_0}{1-rg_0} < \frac{(1-r)\,g_x}{1-rg_x} + \left(1 - \frac{(1-r)\,g_x}{1-rg_x}\right)g_y \tag{C.1}$$

Divide everywhere by $(1-r)$ to get

$$\frac{g_0}{1-rg_0} < \frac{g_x}{1-rg_x} + \frac{g_y}{1-r} - \frac{g_x g_y}{1-rg_x} \tag{C.2}$$

and rearrange to get

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y}{1-rg_x} + \frac{g_y}{1-r} \tag{C.3}$$

Since $0 < g_x < 1$ it follows that

$$\frac{g_y}{1-rg_x} < \frac{g_y}{1-r} \tag{C.4}$$

and it is thus sufficient to prove that

$$\frac{g_0}{1-rg_0} < \frac{g_x - g_x g_y + g_y}{1-rg_x} \tag{C.5}$$

for the proposition to hold.

Now if $g_x > g_0$ we have that

$$\frac{(1-r)\,g_0}{1-rg_0} < \frac{(1-r)\,g_x}{1-rg_x} \tag{C.6}$$

and the proof follows immediately since the term

$$\left(1-\frac{(1-r)\,g_x}{1-rg_x}\right)g_y \tag{C.7}$$

on the right hand side of (C.1) is positive.

On the other hand if $g_x < g_0$ it is sufficient to prove that

$$g_0 < g_x - g_x g_y + g_y \tag{C.8}$$

We know that the two expressions (2.18) and (3.9) must result in the same average first order expectation implying that

$$g_x - g_x g_y + g_y = \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}} \tag{C.9}$$

which can be rearranged to

$$g_x - g_x g_y + g_y = \left(1-\frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}\right) \tag{C.10}$$

From (3.10) we know that

$$g_0 = \left(1-\frac{\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}}\right) \tag{C.11}$$

so that the inequality (C.8) is equivalent to

$$\left(1-\frac{\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma^{-2}}\right) < \left(1-\frac{\gamma^{-1}\sigma^{-2}}{\sigma_\varepsilon^{-2}+\sigma_\eta^{-2}+\gamma^{-1}\sigma^{-2}}\right) \tag{C.12}$$

which was demonstrated to hold generally in the proof of Proposition 3. □

Appendix D. Proof of Proposition 5

Proposition 5 *Holding the precision of the signals and the unconditional variance of x fixed, the response of the average action $\overline{a}$ to a given value of x is stronger if y is available and is a man-bites-dog signal compared to when the signal y is always available.*

Proof. We want to compare the response of $\overline{a}$ described by (3.13) to the response of $\overline{a}^*$ when the signal y is always available while holding the unconditional variance of x and the precision of y fixed. We will call the latter case the *standard signal model* and we will denote quantities and variables associated with the standard signal model with an asterisk (*). The standard signal model is a restricted special case of the set up in Section 3 of the main text. Setting $\omega^* = \gamma^* = 1$ ensures that the signal y is always available. The unconditional variance is held fixed by imposing that $\sigma_x^{*2} = \sigma_x^2$ (and since $\gamma^* = 1$ we also have that $\sigma^{*2} = \sigma_x^{*2}$).

The expression (3.6) for the average action $\overline{a}$ as a function of higher order expectations shows that the average action is increasing in all orders of expectations about x. To prove the proposition, it is thus sufficient to show that all orders of expectation about x respond stronger when y is man-bites-dog signal compared to the case when y is a standard signal of equal precision. The formula (3.9) for the k order expectation in the main text is convenient for deriving the average action as a function of x and y, but is more cumbersome to manipulate for our present purposes. We will therefore derive an alternative (but equivalent) expression for $x^{(k)}$ here.

The average first order expectation of x when y is available is of the form

$$\begin{aligned} x^{(1)} &\equiv \int E\left(x \mid x_j, y\right) dj && \text{(D.1)} \\ &= c_x x + c_y y && \text{(D.2)} \end{aligned}$$

Leaving the coefficients undefined for now, note that substituting in the expression for $x^{(1)}$ into the definition of the second order expectation we get

$$\begin{aligned} x^{(2)} &\equiv \int E\left(x^{(1)} \mid x_j, y\right) dj && \text{(D.3)} \\ &= c_x\left(c_x x + c_y y\right) + c_y y && \text{(D.4)} \end{aligned}$$

and by repeated substitution we get the general expression

$$\begin{aligned} x^{(3)} &= c_x\left(c_x\left(c_x x + c_y y\right) + c_y y\right) + c_y y && \text{(D.5)} \\ &\vdots && \text{(D.6)} \\ x^{(k)} &= c_x^k x + \sum_1^k c_x^{k-1} c_y y && \text{(D.7)} \end{aligned}$$

It is thus sufficient to show that the coefficients c_x and c_y are both larger when y is a man-bites-dog signal compared to when y is a standard signal. Given the parameter restrictions on the standard signal model, the average first order expectation about x is given by

$$x^{*(1)} \equiv \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma^{*-2}} x + \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma^{*-2}} y \tag{D.8}$$

Now, when $\gamma > 1$ and $0 \leq \omega < 1$ the variance σ^{*2} is lower than the conditional variance $\gamma\sigma^2$ in the man-bites-dog model

$$\begin{aligned} \sigma^{*2} &= \sigma_x^2 \\ &= \omega\gamma\sigma^2 + (1-\omega)\sigma^2 \\ &< \gamma\sigma^2 \end{aligned}$$

implying that the coefficients on both x and y in the expression for $x^{*(1)}$ are smaller than the coefficient in the corresponding expression (2.18) for $x^{(1)}$ in the man-bites-dog model

$$\frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma^{*-2}} < \frac{\sigma_\varepsilon^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} \tag{D.9}$$

and

$$\frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \sigma^{*-2}} < \frac{\sigma_\eta^{-2}}{\sigma_\varepsilon^{-2} + \sigma_\eta^{-2} + \gamma^{-1}\sigma^{-2}} \tag{D.10}$$

which completes the proof. □